September 25, 2024

MORE CHAIN RULE

$$f(x) = \cos(3x^2 - 7x)$$
$$f'(x) = -\sin(3x^2 - 7x) \cdot (6x - 7)$$

$$f(x) = \tan^8(x^5 - 3x^4) = [\tan(x^5 - 3x^4)]^8$$
$$f'(x) = 8[\tan(x^5 - 3x^4)]^7 \cdot \sec^2(x^5 - 3x^4) \cdot (5x^4 - 12x^3)$$

$$f(x) = \tan(x^5 - 3x^4)^8$$
$$f'(x) = \sec^2(x^5 - 3x^4)^8 \cdot 8(x^5 - 3x^4)^7 \cdot (5x^4 - 12x^3)$$

$$f(x) = \csc^5(\cot(3x^7))$$

$$f'(x) = 5\csc^4(\cot(3x^7)) \cdot -\csc(\cot(3x^7))\cot(\cot(3x^7)) \cdot -\csc^2(3x^7) \cdot 21x^6$$

$$f(x) = \csc^5(x^2) \cdot \cot(3x^7)$$

$$f'(x) = \csc^5(x^2) \cdot -\csc^2(3x^7) \cdot 21x^6 + \cot(3x^7) \cdot 5\csc^4(x^2) \cdot -\csc(x^2)\cot(x^2) \cdot 2x$$

DIFFERENTIALS

$$y = f(x)$$

$$\frac{dy}{dx} = f'(x)$$

differentials

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

dy ← change in y

dx ← change in x

Find dy.

$$y = x^3 - 3x^2 + 7$$

$$\cancel{dx} \cdot \frac{dy}{\cancel{dx}} = (3x^2 - 6x)\,dx$$

$$dy = (3x^2 - 6x)\,dx$$

The radius of a sphere is measured to be 20 in. With a possible error of ± 0.3 in.

Estimate the possible error in volume. Find dV.

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dr} = 4\pi r^2$

$dV = (4\pi r^2)\,dr$

$dV = (4\pi(20)^2) \cdot \pm 0.3$

$= \pm 150.8 \text{ in}^3$

% error

$\frac{dr}{r} = \frac{\pm 0.3}{20} = 0.015$

1.5%

$\frac{dV}{V} = \frac{\cancel{4\pi} \cancel{r^2}\, dr}{\cancel{\frac{4}{3}\pi} r^{\cancel{3}}} \cdot \cancel{\frac{3}{4}} = \frac{3\,dr}{r}$

$= 3 \cdot \frac{dr}{r}$

$= 3(1.5\%)$

$= 4.5\%$

Dome of a silo $r = 12'$
↑hemisphere

Paint it with a coat of paint 0.002 ft

Estimate the volume of the paint.

Find dV.

$$V = \frac{2}{3}\pi r^3$$

$$dV = 2\pi r^2 \, dr$$

$$dV = 2\pi (\underset{ft}{12})^2 (0.\underset{ft}{002})$$

$$= 1.81 \text{ ft}^3$$