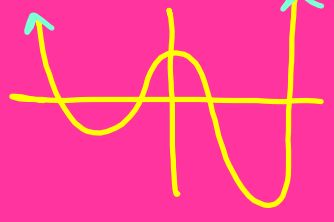
Polynomials & Rational Functions

Polynomials – 1) one or more terms
2) whole # exponents

$1x^4 - 3x^3 + 2x^2 = 7$



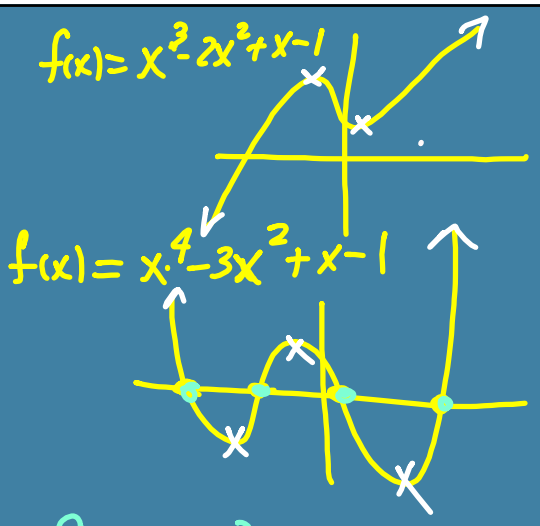
Graph Characteristics

1) Smooth, rounded turns
2) continuous (no asymptotes, no holes, no sharp points, no end pts.)

Degree = Highest power

Max # of relative max/min
Degree − 1

Max # of x-intercepts (zeros)(roots)
Degree



End Behavior

even degree - ends go in same direction
odd degree - ends go in opposite direction

$f(x) = x^2 - 3x + 4$

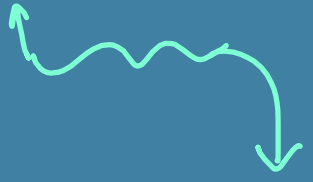
$f(x) = -3x^6 + 7x^5 - 28x^3 + 2x^2 - 7$

both ends go down

$\lim_{x \to \infty} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = -\infty$

$f(x) = -2x^5 + 7x^3 - 4x + 1$



Max of rel max/mins = 5 − 1 = 4
Max of real zeros (x-intercepts) = 5

$\lim_{x \to \infty} f(x) = -\infty$

x goes to right

$\lim_{x \to -\infty} f(x) = +\infty$

x goes to left

Solving Polynomials (Finding roots, zeros, x-int.)

$(2x^3 - 8x^2 + 3x - 12 = 0)$

Algebra
1) Set $x = 0$
2) Factor

$2x^2(x-4) + 3(x-4) = 0$

$(x-4)(2x^2+3) = 0$

$x - 4 = 0$

$x = 4$

$2x^2 + 3 = 0$

$\frac{2x^2}{2} = \frac{-3}{2}$

$\sqrt{x^2} = \sqrt{-\frac{3}{2}}$

$x = \pm \frac{i\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \pm \frac{i\sqrt{6}}{2}$

Solve.

1 $2x^5 + 3x^4 - x^3 + 9x^2 - 55x - 30 = 0$

$(2x+1)(x\ .)(x\ .)(x\ .)(x\ .)$

$\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$ / ± 2

$f(2) = 0$ $(x-2)$ $x = 2$

2	2	3	-1	9	-55	-30
+	0	4	14	26	70	30
	2	7	13	35	15	0

$(x-2)(2x^4 + 7x^3 + 13x^2 + 35x + 15)$

$\pm 1, \pm 3, \pm 5, \pm 15$

Use calculator again

$x = -1$ No

$x = -3$ 0 yes

-3	2	7	13	35	15
		-6	-3	-30	-15
	2	1	10	5	0

$(x-2)(x+3)(2x^3 + x^2 + 10x + 5) = 0$

$(x-2)(x+3)[x^2(2x+1) + 5(2x+1)]$

$(x-2)(x+3)(x^2+5)(2x+1)$

$x = 2$ $x = -3$ $x^2 = -5$ $2x = -1$

$x = \pm i\sqrt{5}$ $x = -1/2$

Steps

1) Find a number that makes $f(x) = 0$ using [factors of last term / factors of leading coeff.]
2) Use # to perform synthetic division
3) Solve remaining factors
4) Repeat, if needed.

Roots: $2, 4, -\frac{3}{2}$

Find eq.

$x = 2$ $\quad x = 4$ $\quad x = -\frac{3}{2}$

$x - 2 = 0$ $\quad x - 4 = 0$ $\quad 2x = -3$

$2x + 3 = 0$

$(x-2)(x-4)(2x+3) = 0$

$(x-2)(2x^2 + 3x - 8x - 12) = 0$

$(x-2)(2x^2 - 5x - 12)$

$2x^3 - 5x^2 - 12x - 4x^2 + 10x + 24$

$2x^3 - 9x^2 - 2x + 24 = 0$