RADICAL EQUATIONS

1) Isolate one root
2) Square both sides

Solve.

$$\sqrt{3x+4} - \sqrt{x+2} = 2$$

$$\left(\sqrt{3x+4}\right)^2 = \left(2 + \sqrt{x+2}\right)^2$$

$(2+\sqrt{x+2})(2+\sqrt{x+2})$ ⟸ FOIL!

$$3x+4 = 4 + 2\sqrt{x+2} + 2\sqrt{x+2} + x + 2$$

$$3x+4 = 6 + x + 4\sqrt{x+2}$$ ← clean up

$$-x \quad -6$$

← Isolate root

$$\left[2x-2\right]^2 = \left[4\sqrt{x+2}\right]^2$$

FOIL! $(2x-2)(2x-2)$

$$4x^2 - 8x + 4 = 16(x+2)$$

$$4x^2 - 8x + 4 = 16x + 32$$

$$-16x - 32$$

$$4x^2 - 24x - 28 = 0$$

$$4(x^2 - 6x - 7) = 0$$

$$4(x-7)(x+1)$$

$x = 7, \cancel{-1}$

↑ check!

Check: $\sqrt{25} - \sqrt{9} = 2$

$5 - 3 = 2$ ✓

$\sqrt{1} - \sqrt{1} = 2$

$1 - 1 = 2$

Partial Fractions

purpose: to find the original fractions that were combined to make the given fraction.

$$\frac{(x-4)}{(x-4)}\frac{2}{x+3} + \frac{6}{x-4}\frac{(x+3)}{(x+3)} = \frac{2x-8+6x+18}{(x+3)(x-4)} = \boxed{\frac{8x+10}{(x+3)(x-4)}}$$

$$\frac{(x-4)}{(x+3)}\left[\frac{8x+10}{x^2-x-12}\right] = \frac{(x+3)}{1}\frac{A}{x-4} + \frac{(x+3)}{(x-4)}\frac{B}{x+3}$$

$$(x-4)(x+3)$$

* multiply by common denom & get rid of all denoms.

$$8x+10 = A(x+3) + B(x-4)$$

$$8x+10 = Ax+3A + Bx-4B$$

$$8 = A+B$$
$$10 = 3A-4B$$

$$\begin{bmatrix} 1 & 1 \\ 3 & -4 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 8 \\ 10 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

$A = 6 \quad B = 2$

$$\boxed{\frac{6}{x-4} + \frac{2}{x+3}}$$

$$\frac{}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x+2}$$

$$\frac{}{(x^2+4)(x^3+7)} = \frac{Ax+B}{x^2+4} + \frac{Cx^2+Dx+E}{x^3+7}$$

$$\frac{}{(x-5)^2(x+3)} = \frac{A}{(x-5)^2} + \frac{B}{(x-5)^1} + \frac{C}{x+3}$$

$$\frac{}{x^3(4x+1)} = \frac{A}{x^3} + \frac{B}{x^2} + \frac{C}{x} + \frac{D}{4x+1}$$

$$\frac{10x^2+24x+8}{(x^3+3x^2+4x+12)} = \frac{\quad}{\quad} + \frac{\quad}{\quad}$$

$x^2(x+3)+4(x+3)$

$(x+3)(x^2+4)$

Get rid of fractions

$$\frac{10x^2+24x+8}{(x+3)(x^2+4)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+4}$$

$$10x^2+24x+8 = A(x^2+4) + (x+3)(Bx+C)$$

$$10x^2+24x+8 = Ax^2+4A + Bx^2+Cx+3Bx+3C$$

Make system of equations

$$10 = A+B+0$$
$$24 = 0+3B+C$$
$$8 = 4A+0+3C$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \\ 4 & 0 & 3 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 10 \\ 24 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ 8 \\ 0 \end{bmatrix}$$

$$\frac{2}{x+3} + \frac{8x}{x^2+4}$$

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