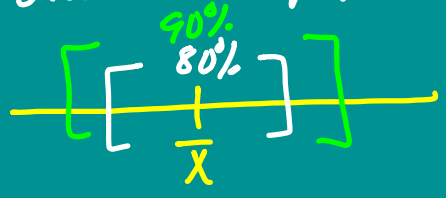
Confidence Intervals

$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$

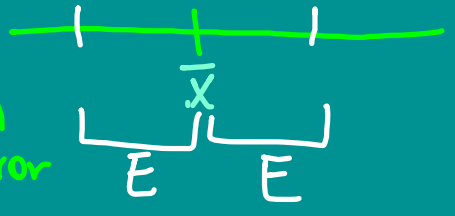
Inferential Statistics – using a sample to describe a population

Confidence Interval –
an interval built around the sample mean ($\bar{x}$) in which the population mean (μ) lies within a certain level of confidence



$\bar{x} \pm E$

↑ margin of error



3 Steps to find a confidence interval.

1) Find standard deviation of the sampling distribution. (standard error of the mean = $\sigma_{\bar{x}}$)

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{s}{\sqrt{n}} \leftarrow \text{if } n \geq 30$$

2) Find margin of error (E)

$$E = z \cdot \sigma_{\bar{x}}$$

z is based on % confident

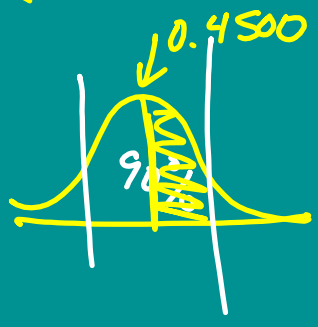
3) Find conf. interval

$$\bar{x} \pm E$$

Mean weight of 36 h.s. wrestlers is 136.4 lb. Standard dev = 14.1 lb. Find a 90% conf. interval for the mean weight of all h.s. wrestlers.

1) Find standard error of the mean.

$$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{14.1}{\sqrt{36}} \approx 2.35$$



2) Find margin of error.

$$E = Z \cdot \sigma_{\bar{x}}$$
$$E = 1.65 \cdot 2.35$$
$$E = 3.88 \text{ lb.}$$

3) $\bar{x} \pm E$

136.4 ± 3.88 lb.

132.52 – 140.28 lb.

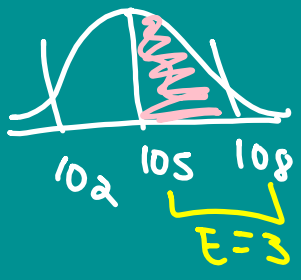
We 90% conf. the mean weight of all h.s. wrestlers falls in this interval

81 cattle fed a special diet

Mean wt. gain = 105 lb. $s = 20$ lb.

What is the probability a cow gained 102 lb - 108 lb.?

probability = % confidence



$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}}$

$= \frac{20}{\sqrt{81}} = 2.227$

$\sigma_{\bar{x}} = \frac{s}{\sqrt{n}}$

$E = z \cdot \sigma_{\bar{x}}$

$E = z \cdot \sigma_{\bar{x}}$

$3 = z \cdot 2.227$

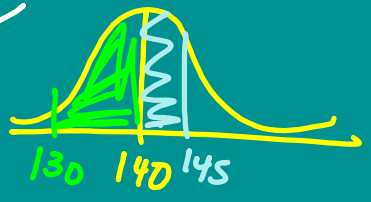
$1.35 = z$

0.4115

x 2

0.8230

Find prob. 130 — 145 $\bar{x} = 140$



$E = z \cdot \sigma_{\bar{x}}$

$10 = z \cdot \#$ $\quad 5 = z \cdot \#$

Sample Size

$$n = \left(\frac{z \cdot \sigma}{E}\right)^2$$

- n: sample #
- z: based on Confidence
- E: margin of Error
- σ: st. dev. of pop.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$E = z \cdot \sigma_{\bar{x}} \Rightarrow \frac{E}{z} = \sigma_{\bar{x}}$$

$$\sqrt{n} \cdot \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \cdot \sqrt{n}$$

$$\sqrt{n} \cdot \sigma_{\bar{x}} = \sigma$$

$$\left(\sqrt{n}\right)^2 = \left(\frac{\sigma}{\sigma_{\bar{x}}}\right)^2$$

$$n = \left(\frac{\sigma}{\frac{E}{z}}\right)^2$$

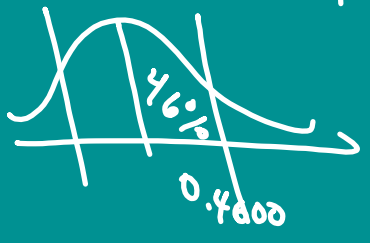
$$n = \left(\frac{\sigma \cdot z}{E}\right)^2$$

What sample size if:

92% prob.

Mean GPA 3.1 - 3.5 (3.3 ± 0.2)

$\sigma = 1.4$



$$n = \left(\frac{1.4 \cdot 1.75}{0.2}\right)^2 = 150.062$$

$\approx$ 151 student