DERIVATIVES OF EXPONENTIAL & LOG FUNCTIONS

8^x, $1.4^x = a^x$, e^x, x^x, $\ln x$, $\log_b x$

$$\boxed{\frac{d}{dx} e^x = e^x}$$

$f(x) = e^x$

$$\lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$$

$$\lim_{h \to 0} \frac{e^x \cdot e^h - e^x}{h}$$

$$\lim_{h \to 0} e^x \frac{(e^h - 1)}{h}$$

$= e^x \cdot 1$

$= e^x$

$f(x) = e^{x^2+4x}$

$f'(x) = e^{x^2+4x} \cdot (2x+4)$

$f(x) = x^2 \cdot e^{4x^3}$

$$f'(x) = \underbrace{x^2 \cdot e^{4x^3} \cdot 12x^2}_{12x^4 e^{4x^3}} + \underbrace{e^{4x^3} \cdot 2x}$$

$= 2xe^{4x^3}\left[6x^3 + 1\right]$

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$y = \ln x$

$e^y = e^{\ln x}$

$e^y = x$

$e^y \cdot \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{e^y}$

$= \frac{1}{x}$

Find $\frac{dy}{dx}$.

$\ln y = \ln a^x$

$\ln y = \ln a^x$

$\ln y = x \cdot \ln a$

$y \cdot \frac{1}{y} \frac{dy}{dx} = \ln a \cdot y$

$\frac{dy}{dx} = y \cdot \ln a$

$= a^x \cdot \ln a$

$\boxed{\frac{d}{dx} \ln x = \frac{1}{x}}$

$\boxed{\frac{d}{dx} a^x = \ln a \cdot a^x}$

$f(x) = 7^x$

$f'(x) = \ln 7 \cdot 7^x$

$f(x) -$

$$f(x) = 5^{x^2}$$
$$f'(x) = \ln 5 \cdot 5^{x^2} \cdot 2x$$
$$= 2\ln 5 \cdot x \cdot 5^{x^2}$$

$$f(x) = 14^{\cos x}$$
$$f'(x) = \ln 14 \cdot 14^{\cos x} \cdot -\sin x$$
$$= -\ln 14 \cdot 14^{\cos x} \cdot \sin x$$

$$f(x) = 6^{\frac{e^{x^3}}{\tan x}}$$
$$f'(x) = \ln 6 \cdot 6^{\frac{e^{x^3}}{\tan x}} \cdot \left[\frac{\tan x \cdot e^{x^3} \cdot 3x^2 - e^{x^3} \cdot \sec^2 x}{\tan^2 x}\right]$$
$$\ln 6 \cdot 6^{\frac{e^{x^3}}{\tan x}} e^{x^3} \left[\frac{3x^2 \tan x - \sec^2 x}{\tan^2 x}\right]$$

Logs

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$f(x) = \ln\left(x^3 - 8x^2\right)$$

$$f'(x) = \frac{1}{x^3 - 8x^2}\cdot\left(3x^2 - 16x\right)$$

$$= \frac{3x^2 - 16x}{x^3 - 8x^2}$$

$$= \frac{\cancel{x}(3x - 16)}{\cancel{x}(x^2 - 8x)}$$

$$= \frac{3x - 16}{x^2 - 8x}$$

$$f(x) = x^3 \cdot \sec(\ln x^2)$$

$$f'(x) = x^{\cancel{3}\,2} \cdot \sec(\ln x^2)\tan(\ln x^2)\cdot\frac{1}{\cancel{x^2}}\cdot 2\cancel{x} + \sec(\ln x^2)\cdot 3x^2$$

$$= 2x^2\sec(\ln x^2)\tan(\ln x^2) + 3x^2\sec(\ln x^2)$$

$$= x^2\sec(\ln x^2)\left[2\tan(\ln x^2) + 3\right]$$

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$$f(x) = \log_8(3x^7)$$

$$= \frac{\ln(3x^7)}{\ln 8}$$

$$= \frac{1}{\ln 8} \cdot \ln(3x^7)$$

$$f'(x) = \frac{1}{\ln 8}\left[\frac{1}{\cancel{3}\cancel{x^7}} \cdot \overset{7}{\cancel{21}}\cancel{x^6}\right]$$

$$= \frac{1}{\ln 8} \cdot \frac{7}{x}$$

<u>Change of Base</u>

$$\log_b a = \frac{\ln a}{\ln b}$$

$$f(x) = x^{x^2}$$
$$= e^{\ln x^{x^2}}$$
$$= e^{x^2 \cdot \ln x}$$
$$f'(x) = \underbrace{e^{x^2 \cdot \ln x}} \cdot \left[x^{\cancel{2}\,1} \cdot \frac{1}{\cancel{x}} + \ln x \cdot 2x \right]$$
$$= x^{x^2} \cdot x^1 \left[1 + 2\ln x \right]$$
$$= x^{x^2+1} \left[1 + 2\ln x \right]$$

Steps for derivative of $x^{f(x)}$ (tower function)

1) Rewrite as $e^{\ln(x^{f(x)})}$
2) Plog to $e^{\boxed{f(x) \cdot \ln x}}$
3) Perform derivative $\left[e^{f(x)\ln x} \cdot \left[\text{product rule} \right] \right]$
4) change $e^{f(x) \cdot \ln x}$ back to the original $x^{f(x)}$
5) Simplify + pull out common factors.

$$f(x) = x^{\cos x} = e^{\ln(x^{\cos x})} = e^{\cos x \cdot \ln x}$$

$$f'(x) = e^{\cos x \cdot \ln x}\left[\cos x \cdot \frac{1}{x} + \ln x \cdot -\sin x\right]$$

$$= x^{\cos x}\left[\frac{\cos x}{x} - \frac{x \cdot \underline{\sin x \ln x}}{x}\right]$$

$$= x^{\cos x}\left[\frac{\cos x - x\sin x \ln x}{x}\right]$$

$$= \frac{x^{\cos x}}{x^1}\left[\cos x - x\sin x \ln x\right]$$

$$= x^{\cos x - 1}\left[\cos x - x\sin x \ln x\right]$$