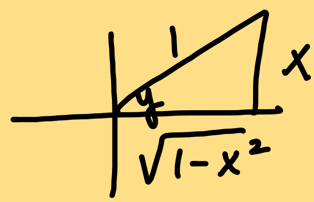
Inverse Trig Functions

$\cancel{\sqrt{9}+\sqrt{16}=\sqrt{25}}$

$3+\cancel{4=5}$

$y = \sin^{-1} x$



$a^2 + x^2 = 1$

$\sqrt{a^2} = \sqrt{1-x^2}$

$[\ \frac{x}{1} = \sin y$

$1 = \cos y \frac{dy}{dx}$

$\frac{1}{\cos y} = \frac{dy}{dx}$

$\frac{x}{r}\quad \frac{1}{\frac{\sqrt{1-x^2}}{1}} = \frac{dy}{dx}$

$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$ $\qquad \frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\tan^{-1}x = \frac{1}{x^2+1}$ $\qquad \frac{d}{dx}\cot^{-1}x = \frac{-1}{x^2+1}$

$\frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}}$ $\qquad \frac{d}{dx}\csc^{-1}x = \frac{-1}{|x|\sqrt{x^2-1}}$

$$f(x) = \sin^{-1}(7x^5) \qquad \frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$

$$f'(x) = \frac{1}{\sqrt{1-(7x^5)^2}} \cdot 35x^4 = \frac{35x^4}{\sqrt{1-49x^{10}}}$$

$$\frac{-1}{|x|\sqrt{x^2-1}}$$

$$f(x) = \csc^{-1}(x^4) \cdot \tan^{-1}(\ln x^2)$$

$$f'(x) = \csc^{-1}(x^4) \cdot \frac{1}{(\ln x^2)^2+1} \cdot \frac{1}{x^{\cancel{2}}} \cdot 2\cancel{x} + \tan^{-1}(\ln x^2) \cdot \frac{-1}{|x^4|\sqrt{x^8-1}} \cdot 4x^3$$

$$= \frac{2\csc^{-1}x^4}{x\left[(\ln x^2)^2+1\right]} + \frac{-4x^3\tan^{-1}(\ln x^2)}{|x^4|\sqrt{x^8-1}}$$

L'Hopital's Rule

$$\lim_{x \to 2} \frac{x^2-4}{x-2} = \frac{0}{0}$$

$$\lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = 4$$

$$\lim_{x \to 2} \frac{2x}{1} = 4$$

Indeterminate Forms

$\frac{0}{0}$, $\frac{\infty}{\infty}$

L'Hopital's Rule

If $\frac{0}{0}$ or $\frac{\infty}{\infty}$

$$\lim_{x \to \#} \frac{f(x)}{g(x)} = \lim_{x \to \#} \frac{f'(x)}{g'(x)}$$

$$\lim_{x \to 1} \frac{x^3-3x^2+5x-3}{x^2+x-2} = \frac{1-3+5-3}{1+1-2} = \frac{0}{0}$$

$$\lim_{x \to 1} \frac{3x^2-6x+5}{2x+1} = \frac{3-6+5}{2+1} = \frac{2}{3}$$

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$$\lim_{x \to 0} \frac{e^x - 1 - x}{\cos(2x) - 1} = \frac{1-1-0}{1-1} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{e^x - 1}{-\sin(2x) \cdot 2} = \frac{1-1}{0 \cdot 2} = \frac{0}{0}$$

$-2\sin(2x)$

$$\lim_{x \to 0} \frac{e^x}{-2\cos(2x) \cdot 2} = \frac{1}{-2 \cdot 1 \cdot 2} = -\frac{1}{4}$$

$\lim\limits_{x\to 0^+} \dfrac{1-\ln x}{e^{1/x}}$ $\quad x^{-1}$ $\quad \lim\limits_{x\to -\infty} e^{x} = 0$ $\quad \lim\limits_{x\to \infty} e^{x} = +\infty$

$\lim\limits_{x\to 0^+} \ln x = -\infty$ $\quad \lim\limits_{x\to \infty} \ln x = +\infty$

$\frac{1}{x} = x^{-1}$ $\quad = \dfrac{1 + +\infty}{e^{1/0}} = \dfrac{\infty}{e^{+\infty}}$

$= \dfrac{\infty}{\infty}$

$\lim\limits_{x\to 0^+} \dfrac{+\frac{1}{x}}{e^{1/x}\cdot + x^{-2}}$ or keep-change-flip

$\lim\limits_{x\to 0^+} \dfrac{+\frac{1}{x}\cdot x^{2}}{e^{1/x}}$

$\lim\limits_{x\to 0^+} \dfrac{x}{e^{1/x}} = \dfrac{0}{e^{1/0^+}} = \dfrac{0}{e^{\infty}} = \dfrac{0}{\infty} = 0$

$\lim\limits_{x\to 0^+} \dfrac{e^{1/x}}{x}$ $\quad \dfrac{+\infty}{+} = \dfrac{+}{0} = +\infty$

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