Special Derivatives

Implicit Differentiation

Multiple variables / cannot solve for y in terms of x

Explicit

$y = 3x^5 - 7x^2 + 4$

$\frac{dy}{dx} = 15x^4 - 14x$

Implicit

$y^2 + 3xy + 7 = 2 - 5y$

$y^2 + x^3 + y^3 = 5$ Find $\frac{dy}{dx}$ ← "normal"

$(3x^5 - 7x^2 + 4)^2 + x^3 + (3x^5 - 7x^2 + 4)^3 = 5$

$2(3x^5 - 7x^2 + 4)^1 \cdot (15x^4 - 14x) + 3x^2 + 3(3x^5 - 7x^2 + 4)^2 \cdot (15x^4 - 14x) = 0$

$2y \cdot \frac{dy}{dx} + 3x^2 + 3y^2 \frac{dy}{dx} = 0$

$2y\frac{dy}{dx} + 3y^2\frac{dy}{dx} = -3x^2$

$\frac{dy}{dx}(2y + 3y^2) = -3x^2$

$\frac{dy}{dx} = \frac{-3x^2}{2y + 3y^2}$

Find $\frac{dy}{dx}$.

$$4y^5 + (3x^2)(y^3) = 6\sin y + 8x^5$$

$$20y^4\frac{dy}{dx} + 3x^2 \cdot 3y^2\frac{dy}{dx} + y^3 \cdot 6x = 6\cos y\frac{dy}{dx} + 40x^4$$

$$20y^4\frac{dy}{dx} + 9x^2y^2\frac{dy}{dx} + 6xy^3 = 6\cos y\frac{dy}{dx} + 40x^4$$

$$\frac{dy}{dx}\left[20y^4 + 9x^2y^2 - 6\cos y\right] = 40x^4 - 6xy^3$$

$$\frac{dy}{dx} = \frac{40x^4 - 6xy^3}{20y^4 + 9x^2y^2 - 6\cos y}$$

Find the equation of the tangent line at $(1,0)$.

$$m = \frac{40(1)^4 - 6(1)(0)^3}{20(0)^4 + 9(1)^2(0)^2 - 6\cos 0} = \frac{40}{-6} = -\frac{20}{3}$$

$$y - 0 = -\frac{20}{3}(x-1)$$

$$\boxed{y = -\frac{20}{3}x + \frac{20}{3}}$$

Find $\frac{dx}{dy}$. (normal →)

$$\frac{x^2}{y} = 4y^3 + 6x$$

$$y^2\left[\frac{y \cdot 2x\frac{dx}{dy} - x^2 \cdot 1}{y^2} = 12y^2 + 6\frac{dx}{dy}\right]$$

$$2xy\frac{dx}{dy} - x^2 = 12y^4 + 6y^2\frac{dx}{dy}$$

$$\frac{dx}{dy}\left[2xy - 6y^2\right] = 12y^4 + x^2$$

$$\frac{dx}{dy} = \frac{12y^4 + x^2}{2xy - 6y^2}$$

Find $\frac{da}{dp}$. $3r^7 + 6a^5 - 4p = p^7$

normal →

$$21r^6\frac{dr}{dp} + 30a^4\frac{da}{dp} - 4 = 7p^6$$

$$\frac{30a^4\frac{da}{dp}}{30a^4} = \frac{7p^6 - 21r^6\frac{dr}{dp} + 4}{30a^4}$$

Find $\frac{dy}{dt}$. $4x^2 + 2y^5 = \cos x$

normal →

$$8x\frac{dx}{dt} + 10y^4\frac{dy}{dt} = -\sin x\frac{dx}{dt}$$

$\frac{\Delta y}{\Delta t}$

$$10y^4\frac{dy}{dt} = -8x\frac{dx}{dt} - \sin x\frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{(-8x - \sin x)\frac{dx}{dt}}{10y^4}$$

13/ $\sin(xy) = x + y$

$\cos(xy) \cdot \left[x \cdot 1\frac{dy}{dx} + y \cdot 1 \right] = 1 + 1\frac{dy}{dx}$

$x\cos(xy)\frac{dy}{dx} + y\cos(xy) = 1 + \frac{dy}{dx}$

$x\cos(xy)\frac{dy}{dx} - \frac{dy}{dx} = 1 - y\cos(xy)$

$\frac{dy}{dx}\left(x\cos(xy) - 1\right)$

SQUARE ROOTS

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

$\sqrt{4} = 2$

$\frac{3(x-4)^2}{3} = \frac{48}{3}$

$\sqrt{(x-4)^2} = \sqrt{16}$

$x - 4 = \pm 4$

$x = 4 \pm 4$

$x = 8$ or 0

$x^2 + 8 = 20$

$\sqrt{x^2} = \sqrt{12}$

$4 \cdot 3$

$x = \pm 2\sqrt{3}$

$x = 4 \pm \sqrt{3}$

$\sqrt{-9} = \sqrt{-1 \cdot 9}$

$= 3i$

$\sqrt{-1} = i$

$\sqrt{-24}$

$4 \cdot 6 \cdot -1$

$= 2i\sqrt{6}$