

# SPECIAL DERIVATIVES

## Implicit Differentiation

Multiple Variables  
cannot solve for  
y in terms of x

Explicit

$$y = 3x^5 - 7x^2 + 4$$

$$\frac{dy}{dx} = 15x^4 - 14x$$

Implicit

$$y^2 + 3xy + 7 = 2 - 5y$$

$$y^2 + x^3 + y^3 = 5$$

Find  $\frac{dy}{dx}$  ← "normal"

$$(3x^5 - 7x^2 + 4)^2 + x^3 + (3x^5 - 7x^2 + 4)^3 = 5$$

$$2(3x^5 - 7x^2 + 4) \cdot (15x^4 - 14x) + 3x^2 + 3(3x^5 - 7x^2 + 4)^2 \cdot (15x^4 - 14x) = 0$$

$$2y \cdot \frac{dy}{dx}$$

$$+ 3x^2 + 3y^2 \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = -3x^2$$

$$\frac{dy}{dx} (2y + 3y^2) = -3x^2$$

$$\frac{dy}{dx} = \frac{-3x^2}{2y + 3y^2}$$

Find  $\frac{dy}{dx}$ .

$$4y^5 + (3x^2y^3) = 6\sin y + 8x^5$$

$$20y^4 \frac{dy}{dx} + 3x^2 \cdot 3y^2 \frac{dy}{dx} + y^3 \cdot 6x = 6\cos y \frac{dy}{dx} + 40x^4$$

$$20y^4 \frac{dy}{dx} + 9x^2y^2 \frac{dy}{dx} + 6xy^3 = 6\cos y \frac{dy}{dx} + 40x^4$$

$$\frac{dy}{dx} [20y^4 + 9x^2y^2 - 6\cos y] = 40x^4 - 6xy^3$$

$$\frac{dy}{dx} = \frac{40x^4 - 6xy^3}{20y^4 + 9x^2y^2 - 6\cos y}$$

Find the equation of the tangent line at  $(1, 0)$ .

$$m = \frac{40(1)^4 - 6(1)(0)^3}{20(0)^4 + 9(1)^2(0)^2 - 6\cos 0} = \frac{40}{-6} = -\frac{20}{3}$$

$$y - 0 = -\frac{20}{3}(x - 1)$$

$$\boxed{y = -\frac{20}{3}x + \frac{20}{3}}$$

Find  $\frac{dx}{dy}$ .  $\frac{x^2}{y} = 4y^3 + 6x$   
 normal  $\rightarrow$

$$\cancel{y^2} \left[ \frac{y \cdot 2x \frac{dx}{dy} - x^2 \cdot 1}{\cancel{y^2}} = 12y^2 + 6 \frac{dx}{dy} \right]$$

$$2xy \frac{dx}{dy} - x^2 = 12y^4 + 6y^2 \frac{dx}{dy}$$

$$\frac{dx}{dy} [2xy - 6y^2] = 12y^4 + x^2$$

$$\frac{dx}{dy} = \frac{12y^4 + x^2}{2xy - 6y^2}$$

Find  $\frac{da}{dp}$ .  $3r^7 + 6a^5 - 4p = p^7$

normal  $\rightarrow$

$$21r^6 \frac{dr}{dp} + 30a^4 \frac{da}{dp} - 4 = 7p^6$$

$$\frac{30a^4 \frac{da}{dp}}{30a^4} = \frac{7p^6 - 21r^6 \frac{dr}{dp} + 4}{30a^4}$$

Find  $\frac{dy}{dt}$ .  
normal  $\rightarrow$

$$\frac{\Delta y}{\Delta t}$$

$$4x^2 + 2y^5 = \cos x$$

$$8x \frac{dx}{dt} + 10y^4 \frac{dy}{dt} = -\sin x \frac{dx}{dt}$$

$$10y^4 \frac{dy}{dt} = -8x \frac{dx}{dt} - \sin x \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{(-8x - \sin x) \frac{dx}{dt}}{10y^4}$$

$$\begin{aligned} 13/ \quad \sin(xy) &= x + y \\ \cos(xy) \cdot \left[ \underset{\curvearrowright}{x \cdot \frac{dy}{dx}} + \underset{\curvearrowright}{y \cdot 1} \right] &= 1 + \frac{dy}{dx} \\ x \cos(xy) \frac{dy}{dx} + y \cos(xy) &= 1 + \frac{dy}{dx} \\ x \cos(xy) \frac{dy}{dx} - \frac{dy}{dx} &= 1 - y \cos(xy) \\ \frac{dy}{dx} (x \cos(xy) - 1) & \end{aligned}$$

# SQUARE ROOTS

$$\sqrt{x^2} = \sqrt{4}$$

$$x = \pm 2$$

$$\sqrt{4} = 2$$

$$\frac{3(x-4)^2}{3} = \frac{48}{3}$$

$$\sqrt{(x-4)^2} = \sqrt{16}$$

$$x-4 = \pm 4$$

$$x = 4 \pm 4$$

$$\boxed{x = 8 \text{ or } 0}$$

$$x^2 + 8 = 20$$

$$\sqrt{x^2} = \sqrt{12}$$

4.3

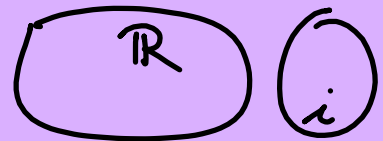
$$x = \pm 2\sqrt{3}$$

$$x = 4 \pm \sqrt{3}$$

$$\sqrt{-9} = \sqrt{-1 \cdot 9}$$

$$= 3i$$

$$\sqrt{-1} = i$$



$$\sqrt{-24} = 2i\sqrt{6}$$

4.6.1