

# LOGARITHMS - John Napier 1614

Exp  
form  $y = b^x$   $b > 0, b \neq 1$

$x = b^y$  Domain:  $(-\infty, \infty)$   
Range:  $(0, \infty)$

$\log_b x = y$  Domain:  $(0, \infty)$   
Range:  $(-\infty, \infty)$

Logarithms are  
inverses of exponential  
functions.

Logs are used  
to solve for  
exponents!

$$2^3 = 8$$

$$\log_{12} 144 = \log_{12} 12^2 = 2$$

$$\log_2 16 = \log_2 2^4 = 4$$

$$\log_9 \frac{1}{81} = \log_9 \frac{1}{9^2} = \log_9 9^{-2} = -2$$

$$\log_7 \sqrt[5]{7} = \log_7 7^{1/5} = \frac{1}{5}$$

$$\log_{11} \sqrt[3]{\frac{1}{121}} = \log_{11} \sqrt[3]{\frac{1}{11^2}} = \log_{11} 11^{-2/3} = -\frac{2}{3}$$

Make  
common  
bases!

## Special Logs

### Common logs

$$\log_{10} x = \log x$$

### Natural logs

$$\log_e x = \ln x$$

$$\log_{10} 1000 = \log_{10} 10^3$$

$$= 3$$

$$\ln \sqrt[5]{e^3} = \ln e^{3/5} = \frac{3}{5}$$

$$8^{\log_8 50} = 50$$

$$e^{\ln 17} = 17$$

$$y = 3^{x-4} + 2$$

$$y = \log_3(x-4) + 2$$

Right UP  
2

$$\begin{array}{r|l} 0 & 1 \\ 1 & 3 \\ 2 & 9 \end{array}$$

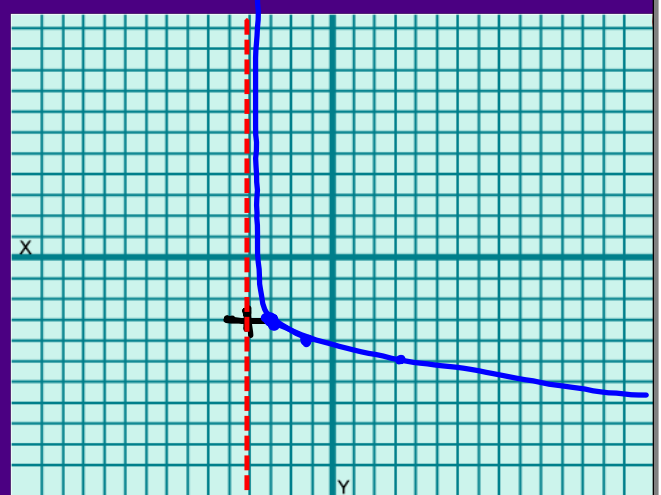
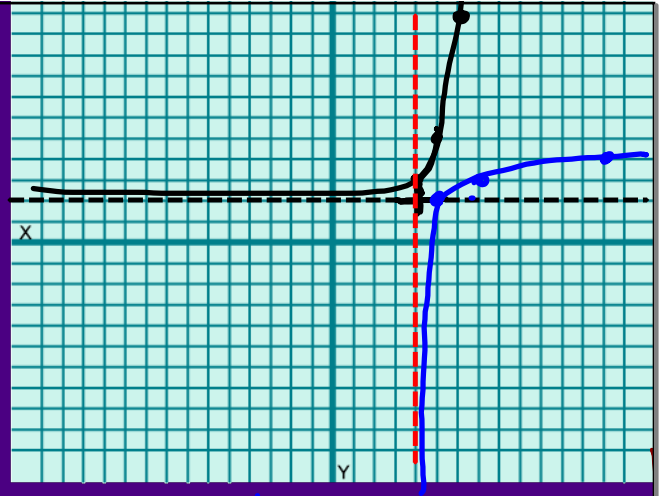
$$\begin{array}{r|l} 1 & 0 \\ 3 & 1 \\ 9 & 2 \end{array}$$

$$y = -\ln_e(x+4) - 3$$

$$\begin{array}{r|l} 1 & 0 \\ 2.7 & -1 \\ 7.4 & -2 \end{array}$$

0

$$y = \log_2(x)$$



# SOLVING LOG EQUATIONS

Properties

$$\log_b m + \log_b n = \log_b (m \cdot n)$$

$$\log_b m - \log_b n = \log_b \left( \frac{m}{n} \right)$$

$$\log_b m^p = p \cdot \log_b m$$

$$\log_x 64 = 3$$

$$x^{\log_x 64} = x^3$$

$$\sqrt[3]{64} = \sqrt[3]{x^3}$$

$$\boxed{4 = x}$$

Exponentiate!

$$\log_7 (x+5) + \log_7 (x-3) = 2 \log_7 3$$

$$\log_7 (x+5)(x-3) = \log_7 3^2$$

$$\log_7 (x^2 + 2x - 15) = \log_7 9$$

$$\log_7 (x^2 + 2x - 15) = \log_7 9$$

$$x^2 + 2x - 15 = 9$$

$$x^2 + 2x - 24 = 0$$

$$(x+6)(x-4) = 0$$

$$x = -\cancel{6}, 4$$

$$\log(x+3) - \log x = 1$$

$$\log\left(\frac{x+3}{x}\right) = 1$$

$$\frac{x+3}{x} = 10x$$

$$x+3 = 10x$$

$$3 = 9x$$

$$\frac{3}{9} = x$$

$$\boxed{\frac{1}{3} = x}$$