More L'Hopital's Rule

$$\lim_{x\to 0^+} x^2 \cdot \ln x = 0 \cdot -\infty$$

$$\lim_{x\to 0^+} \frac{\ln x}{x^{-2}} = \frac{-\infty}{\infty}$$

Must be in fraction form!

$$\lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-2}{x^3}} \cdot \frac{x^3}{-2}$$

$$\lim_{x\to 0^+} \frac{x^2}{-2} = \frac{0}{-2} = \boxed{0}$$

Indeterminate Forms

$\frac{0}{0}$, $\frac{\infty}{\infty}$

$0 \cdot \infty$, $\infty - \infty$

0^0, 1^∞, ∞^0

$\cancel{0^\infty}$

Ways to rearrange into fraction form:

* Move x to denom with negative power
* Use fundamental trig identities

$$\lim_{x\to 0^+}\left(\csc x - \frac{1}{x}\right) = \infty - \infty$$

$\frac{1}{\sin x}$

$$\lim_{x\to 0^+}\left(\frac{x\cdot 1}{x\cdot \sin(x)} - \frac{1\cdot \sin x}{x\cdot \sin x}\right)$$

$$\lim_{x\to 0^+}\frac{x-\sin x}{x\cdot \sin x} = \frac{0-0}{0\cdot 0}$$

$$\lim_{x\to 0^+}\frac{1-\cos x}{x\cdot \cos x + \sin x\cdot 1} = \frac{1-1}{0\cdot 1 + 0\cdot 1}$$

$$\lim_{x\to 0^+}\frac{+\sin x}{x\cdot -\sin x + \cos x\cdot 1 + \cos x}$$

$$\lim_{x\to 0^+}\frac{\sin x}{-x\sin x + 2\cos x} = \frac{0}{0\cdot 0 + 2(1)}$$

$$= \frac{0}{2}$$

$$= 0$$

$$\lim_{x\to\infty} x^{1/x} = \infty^{1/\infty} = \infty^0$$

$$\lim_{x\to\infty} e^{\ln x^{1/x}}$$

$$\lim_{x\to\infty} \frac{1}{x}\cdot \ln x$$

$$\lim_{x\to\infty} \frac{\ln x}{x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \frac{1}{\infty} = 0$$

$$e^0 = \boxed{1}$$

$$\lim_{x\to 0^+} (\csc x)^{\sin x} = \infty^0$$

$$\lim_{x\to 0^+} e^{\ln(\csc x)^{\sin x}}$$

$$e^{\lim_{x\to 0^+} \sin x \cdot \ln(\csc x)}$$

$$\lim_{x\to 0^+} \frac{\ln(\csc x)}{\csc x} = \frac{\infty}{\infty}$$

$$\lim_{x\to 0^+} \frac{\frac{1}{\csc x}\cdot \cancel{-\csc x \cot x}}{\cancel{-\csc x \cot x}}$$

$$= \frac{1}{\infty} = 0$$

$$\boxed{e^0 = 1}$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} = (1+0)^{\infty} = 1^{\infty}$$

$$\lim_{x\to\infty} e^{x\cdot\ln\left(1+\frac{1}{x}\right)} \qquad\qquad e^{\ln(f(x))}$$

$$e^{\lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{x^{-1}}} = \frac{0}{0}$$

$$\frac{1}{\infty}$$

$$\lim_{x\to\infty}\frac{\frac{1}{1+\frac{1}{x}}\cdot\cancel{-\frac{1}{x^{2}}}}{\cancel{-\frac{1}{x^{2}}}}$$

$$= \frac{1}{1+\frac{1}{\infty}} = \frac{1}{1+0} = 1$$

$$e^{1}$$