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Natural Log Operations

$$\ln x + \ln(x+3) = 2$$

$$e^{\ln(x^2+3x)} = e^2$$

$$x^2+3x = e^2$$

$$x^2+3x-e^2 = 0$$

$$x = \frac{-3 \pm \sqrt{3^2-4(1)(-e^2)}}{2(1)}$$

$$\approx 1.605,\ -\cancel{4.605}$$

$$\frac{2e^{2x-5}}{2} = \frac{32}{2}$$

$$\ln e^{2x-5} = \ln(16)$$

$$2x-5 = \ln(16)$$

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{\ln(16)+5}{2}$$

$$\approx 3.886$$

$$4^{2x+3} = 7^{x-1}$$

$$\log 4^{2x+3} = \log 7^{x-1}$$

$$(2x+3)\log 4 = (x-1)\log 7$$

$$2x\log(4) + 3\log(4) = x\log 7 - \log 7$$

$$2x\log(4) - x\log(7) = -\log(7) - 3\log(4)$$

$$x\left(2\log(4) - \log(7)\right) = -\log(7) - 3\log(4)$$

$$x = \frac{-\log(7) - 3\log(4)}{2\log(4) - \log(7)}$$

$$x \approx -7.385$$

$$e^{2x} + 3e^{x} = 28$$

$$e^{2x} + 3e^{x} - 28 = 0$$

$$(e^{x}+7)(e^{x}-4) = 0$$

$e^{x}+7=0$ $\quad$ $e^{x}-4=0$

$\ln e^{x} = \ln -7$ $\quad$ $\ln e^{x} = \ln 4$

$x = \cancel{\ln(-7)}$ $\quad$ $x = \ln(4) \approx 1.386$

Radioactive Iodine has a half-life of 60 days. It is considered to be safe when 5% or less is left. How many days will it take to reach a safe level.

$N = N_0 e^{kt}$

$0.5 = 1\, e^{k \cdot 60}$

$\ln 0.5 = \ln e^{60k}$

$\frac{\ln(0.5)}{60} = \frac{60k}{60}$

$-0.0116 = k$

$0.05 = 1\, e^{-0.0116t}$

$\ln 0.05 = \ln e^{-0.0116t}$

$\frac{\ln(0.05)}{-0.0116} = \frac{\cancel{-0.0116}t}{\cancel{-0.0116}}$

$258 \text{ days} = t$

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Newton's Law of Cooling

$$u = \underset{\text{Air}}{T} + (u_0 - \underset{\text{Air}}{T})e^{kt}$$

Room Temp = 71°

Normal body temp = 98.6°

Body found = 75°

1 hr. later = 72°

$$72 = 71 + (75 - 71)e^{K \cdot 1}$$

$$72 = 71 + 4e^{K}$$

$$-71$$

$$1 = 4e^{K}$$

$$\ln(0.25) = \ln(e^{K})$$

$$\ln(0,25) = K$$

$$-1.386 = K$$

$$75 = 71 + (98.6 - 71)e^{-1.386t}$$

$$75 = 71 + 27.6e^{-1.386t}$$

$$-71$$

$$\frac{4}{27.6} = \frac{27.6e^{-1.386t}}{27.6}$$

$$\ln\frac{4}{27.6} = \ln e^{-1.386t}$$

$$\frac{\ln\left(\frac{4}{27.6}\right)}{-1.386} = \frac{-1.386t}{-1.386}$$

$$1.39 \text{ hrs.} = t$$