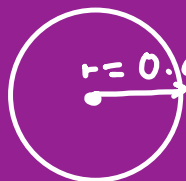


RELATED RATES

$$\frac{\text{mi}}{\text{h}} \quad \frac{\text{m}}{\text{s}} \quad \frac{\text{cm}}{\text{h}} \quad \frac{\text{gal}}{\text{min}} \quad \frac{\text{rad}}{\text{sec}}$$

- rate of one part of the situation impacts the rate of another part.

Example 1



$$\frac{d}{dt} [A = \pi r^2]$$

~~$2\pi r$~~

$$1 \cdot \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi \left(\frac{4}{\text{in}} \right) \left(0.02 \frac{\text{in}}{\text{sec}} \right)$$

1) Draw a picture

2) Label with variables (changing) & constants (not changing)

$$\frac{dA}{dt} = 0.16\pi \frac{\text{in}^2}{\text{sec}}$$

3) Set up a formula

$$\approx \boxed{0.5 \frac{\text{in}^2}{\text{s}}}$$

4) Do derivative with respect to time using implicit differentiation.

5) Identify the rate to be found.

6) Fill in values & solve.

$$2. \frac{d}{dt} \left[V = \frac{4}{3} \pi r^3 \right]$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$-0.2 \frac{\text{m}^3}{\text{min}} = 4\pi (0.4 \text{ m})^2 \frac{dr}{dt}$$

$$\frac{-0.2}{0.64\pi} = \frac{0.64\pi}{0.64\pi} \frac{dr}{dt}$$

$$-0.0994 = \frac{dr}{dt}$$

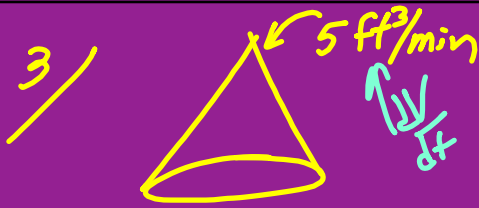
$$\approx \boxed{-0.1 \frac{\text{m}}{\text{min}}} = \frac{dr}{dt}$$

$$S.A = 0.64\pi$$

$$\frac{4\pi r^2}{4\pi} = \frac{0.64\pi}{4\pi}$$

$$\sqrt{r^2} = \sqrt{0.16}$$

$$r = 0.4$$



$h = 2r \Rightarrow \frac{h}{2} = r$
 Find rate of height
 when 10 ft high

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{h}{2}\right)^2 h$$

$$V = \frac{1}{3} \pi \frac{h^2}{4} \cdot h$$

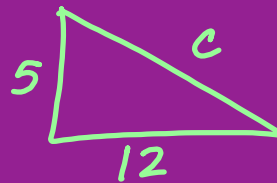
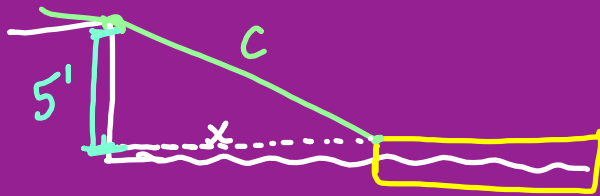
$$\frac{d}{dt} \left[V = \frac{1}{12} \pi h^3 \right]$$

$$\frac{dV}{dt} = \frac{1}{4} \pi h^2 \frac{dh}{dt}$$

$$\frac{5 \frac{\text{ft}^3}{\text{min}}}{\frac{\text{ft}^2}{\text{ft}^2}} = \frac{1}{4} \pi (10)^2 \frac{dh}{dt}$$

$$\frac{5}{25\pi} = \frac{25\pi \frac{dh}{dt}}{25\pi}$$

$$\frac{1}{5\pi} \frac{\text{ft}}{\text{min}} = \frac{dh}{dt}$$



$$\frac{d}{dt} [c^2 = 25 + x^2]$$

$$2c \frac{dc}{dt} = 2x \frac{dx}{dt}$$

$$2(13) \underset{\text{ft}}{\underset{\text{ft/min}}{(-4)}} = 2(12) \underset{\text{ft}}{\frac{dx}{dt}}$$

$$\frac{-104}{24} = \frac{24 \frac{dx}{dt}}{24}$$

$$-\frac{13}{3} \text{ ft/min} = dx/dt$$

$$5^2 + 12^2 = c^2$$

$$169 = c^2$$

$$13 = c$$