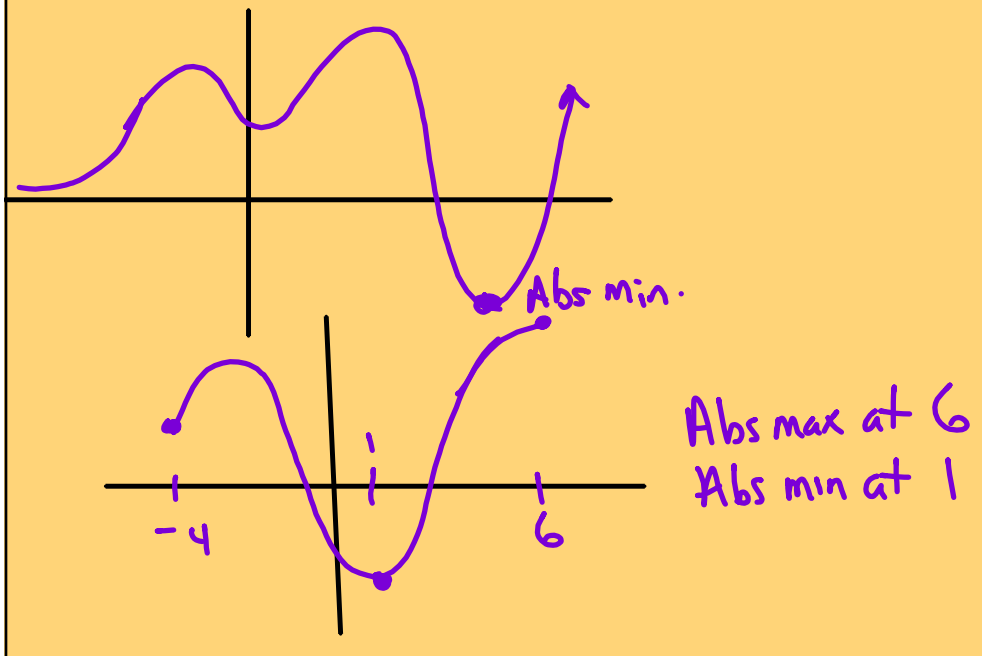
November 20, 2024

ABSOLUTE EXTREMA



Abs max at 6
Abs min at 1

$f(x) = x^2 - 3x + 2$ $\quad [0, 5]$

$f'(x) = 2x - 3 = 0$

$x = 3/2$

x	y
0	2
3/2	-1/4
5	12

Abs max at $(5, 12)$

Abs min at $(3/2, -1/4)$

1) Find crit pts.
2) Sub crit pts. + end points in f to find highest & lowest values.

$f(x) = 3 - 4x - 2x^2 \quad (-\infty, \infty)$

$\lim_{x \to -\infty} 3 - 4x - 2x^2 = \lim_{x \to -\infty} -2x^2 = -2(-\infty)^2 = -\infty$

$\lim_{x \to \infty} -2x^2 = -\infty$

No Abs min

Abs max (-1, 5)

$f'(x) = -4 - 4x = 0$

$-4 = 4x$

$-1 = x$

-1 | 5

If open interval (—, —)

1) Check limits of endpoints.
2) Find crit pts
3) Find y-coord of crit pts & compare to limits

$f(x) = (x^3-1)^{2/3} \quad (-1, 4]$

1) $\lim_{x \to -1^+} (x^3-1)^{2/3} = ((-1)^3-1)^{2/3}$

$= (-2)^{2/3}$

$= \sqrt[3]{(-2)^2}$

$= \sqrt[3]{4} \approx 1.587$

0	$-1^{2/3} = 1$
1	0
4	$63^{2/3} = \sqrt[3]{3969}$

≈ 15.93

Abs max $(4, 63^{2/3})$

Abs min $(1, 0)$

2) Crit pts.

$f'(x) = \frac{2}{3}(x^3-1)^{-1/3} \cdot 3x^2$

$= \frac{2x^2}{\sqrt[3]{x^3-1}} = 0$

$2x^2 = 0$

$x = 0 \quad x = 1$

$f(x) = \frac{x}{x^2+1} \quad (0, \infty)$

$\lim_{x \to 0^+} \frac{x}{x^2+1} = \frac{0}{1} = 0$

$\lim_{x \to \infty} \frac{x}{x^2+1} = \frac{\infty}{\infty}$

$\lim_{x \to \infty} \frac{1}{2x} = \frac{1}{\infty} = 0$

1	1/2

Abs max $(1, 1/2)$
No Abs min

$f'(x) = \frac{(x^2+1)1 - x(2x)}{(x^2+1)^2}$

$= \frac{x^2+1-2x^2}{(x^2+1)^2}$

$= \frac{1-x^2}{(x^2+1)^2} = 0$

$1 - x^2 = 0$

$\sqrt{1} = \sqrt{x^2}$

$\pm 1 = x$

not in interval

$f(x) = x^2 e^{2x} \qquad [-2, \infty)$

$\lim_{x \to \infty} x^2 e^{2x} = \infty \cdot \infty = \infty$

No Abs max
Abs min (0,0)

$f'(x) = x^2 \cdot e^{2x} \cdot 2 + e^{2x} \cdot 2x$

$2xe^{2x}\left[x + 1\right]$

$2x = 0 \qquad x = -1$

$x = 0$

-1	$1e^{-2} = \frac{1}{e^2} \approx 0.135$
0	$0 \cdot e^0 = 0$
-2	$4 \cdot e^{-4} = \frac{4}{e^4} =$