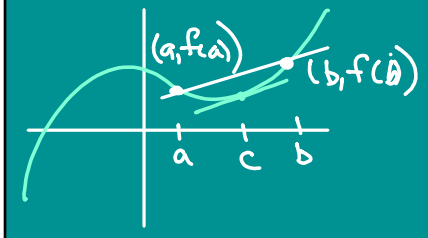
CURVE SKETCHING

Mean Value Theorem for Derivatives



1) f is continuous on $[a,b]$
2) f is differentiable on (a,b)

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

Find point c.

$f(x) = x^3 - 3x^2 + 2x$ on $[a,b] = [0,2]$

$f'(x) = 3x^2 - 6x + 2$

$$3c^2 - 6c + 2 = \frac{f(2)-f(0)}{2-0}$$

$$3c^2 - 6c + 2 = \frac{(8-12+4)-(0-0+0)}{2}$$

$$= \frac{0-0}{2}$$

$$3c^2 - 6c + 2 = 0$$

$$c = \frac{6 \pm \sqrt{36-4(3)(2)}}{2(3)}$$

$$c = \frac{6 \pm \sqrt{12}}{6}$$

$$c = \frac{6 \pm 2\sqrt{3}}{6}$$

$$c = \frac{3 \pm \sqrt{3}}{3}$$

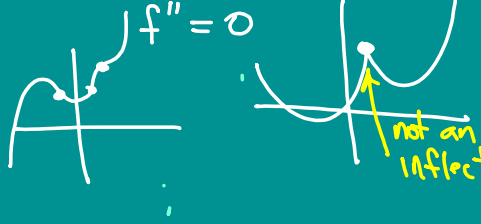
$c \approx 1.6$ $c \approx 0.4$

Critical points:
1) where $f' = 0$
or
2) where f' is undefined but f is defined.

← Points of non-differentiability

Inflection points = where concavity changes

$f'' = 0$



$f(x) = x^4 - 4x^3 + 10$

1) Find critical pts.

$f'(x) = 0$

$f'(x) = 4x^3 - 12x^2 = 0$

$\rightarrow 4x^2(x-3) = 0$

$\sqrt{x^2} = \sqrt{0}$ $\quad x - 3 = 0$

$x = 0$ $\quad x = 3$

f' $\quad +\cdot- \quad +\cdot- \quad +\cdot+$

$\quad\quad - \quad\quad - \quad\quad +$

$\quad -1 \quad 0 \quad 1 \quad 3 \quad 1000$

2) Infl pts. + Concavity

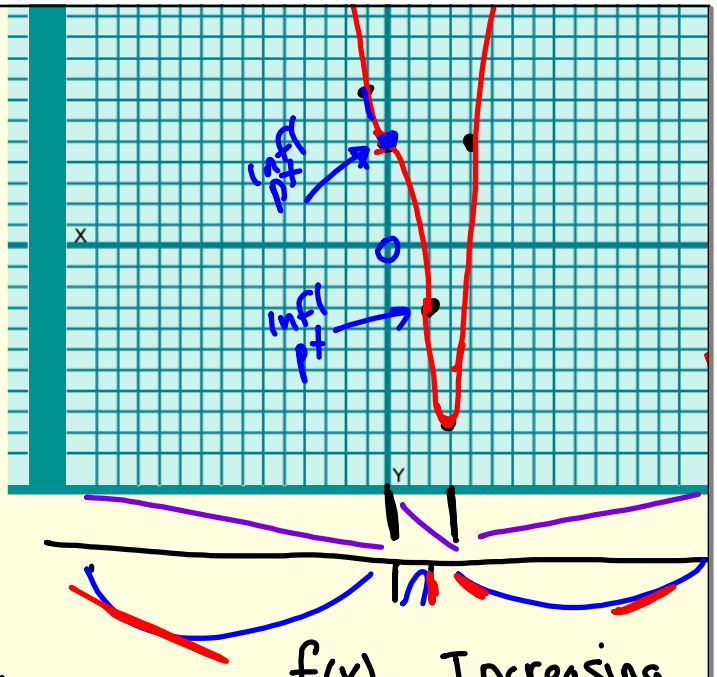
$f'' = 12x^2 - 24x$

$\Rightarrow 0 = 12x(x-2)$

$x = 0 \quad x = 2$

$+ \quad +\cdot- \quad +$

$0 \quad 2$



x	f(x)
0	10
2	-6
3	-17
-1	15
4	10

Increasing Intervals

$(3, \infty)$

Decreasing

$(-\infty, 0)$

$(0, 3)$

$f(x) = (x-1)^{1/2}$

$f'(x) = \frac{1}{2}(x-1)^{-1/2} \cdot 1$

$= \frac{1}{2\sqrt{x-1}} \leftarrow x \neq 1$

$x = 1 \leftarrow$ pt. of non-differentiability

$f(x) = 3x^{2/3} - x$

$f'(x) = 2x^{-1/3} - 1 = 0$

$\frac{2}{\sqrt[3]{x}} = 1 \cdot \sqrt[3]{x}$

$(2)^3 = (\sqrt[3]{x})^3$

$8 = x$

$0 = x$

$f'(x) = \frac{2}{\sqrt[3]{x}} - 1 = 2x^{-1/3} - 1$

$-2-1 \quad 2-1 \quad \frac{3}{3}-1$

$- \; | \; + \; | \; -$

$-1 \quad 0 \quad 1 \quad 8 \quad 27$

$f''(x) = -\frac{2}{3}x^{-4/3} = 0$

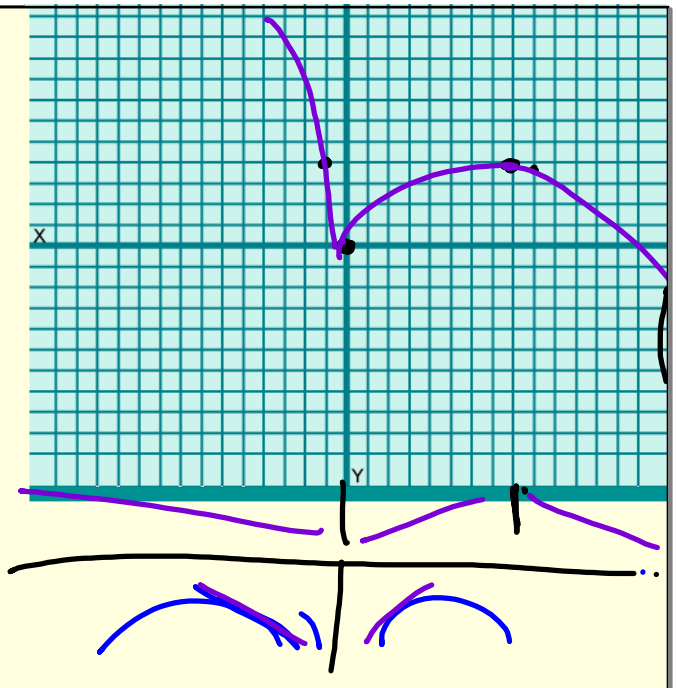
$-\frac{2}{3\sqrt[3]{x^4}} = 0$

$2 = 0$

Undef. $x = 0$

$- \; | \; -$

0



$x =$	
0	0
8	4
-1	4
9	$\simeq 3.98$

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