CALCULUS SEMESTER REVIEW

LIMITS

- $x \to \#$
  - Sub # in
    - $\frac{0}{0}$
      - factor
      - conjugate
      - L'Hop.
    - $\#/0$
      - one-sided limits ($\pm\infty$)
- $x \to \pm\infty$
  - $\frac{\infty}{\infty}$
    - Use highest power terms or L'Hop
  - poly/poly
    - Use highest power term
- special limits
  - e^x, $\ln x$, 7^x, x^x
  - L'Hop.
  - Indeterminate
    - ∞^0, 0^0, 1^∞
      - $e^{\ln f(x)}$
    - $0 \cdot \infty$, $\infty - \infty$
      - rearrange to fraction

1(e ii)

$$\lim_{x \to -\infty} \frac{5-6x^3}{\sqrt[2]{4x^6-3x^4+2}}$$

$$\lim_{x \to -\infty} \frac{-6x^3}{\sqrt{4x^6}}$$

$$\lim_{x \to -\infty} \frac{-6x^3}{2|x^3|}$$

$$\lim_{x \to -\infty} \frac{-6\cancel{x^3}}{-2\cancel{x^3}} = 3$$

$$\lim_{x \to 2^+} (x-1)^{\frac{1}{x-2}}$$

$$= 1^{\frac{1}{0}}$$

$$= 1^{+\infty}$$

$$\lim_{x \to 2^+} e^{\ln (x-1)^{1/x-2}}$$

$$e^{\lim_{x \to 2^+} \frac{1}{x-2} \cdot \ln(x-1)}$$

$$\lim_{x \to 2^+} \frac{\ln(x-1)}{x-2} = \frac{0}{0}$$

$$\lim_{x \to 2^+} \frac{\frac{1}{x-1} \cdot 1}{1} = \frac{1}{2-1} = 1$$

$$= \boxed{e^1}$$

13/17 Continuity / Differentiability

$$f(x) = \begin{cases} 2x^2 - 4x + 1 & x \le 1 \\ \frac{1}{-x} & x > 1 \end{cases} \qquad a = 1$$

1) $f(1) = 2(1)^2 - 4(1) + 1 = -1$

2) $\lim_{x \to 1^-} (2x^2 - 4x + 1) = -1$

$\lim_{x \to 1^+} -\frac{1}{x} = -1$

$\lim_{x \to 1} f(x) = -1$

3) $f(1) = \lim_{x \to 1} f(x)$

Continuous

4) $f'(1)^- = 4x - 4 = 4(1) - 4 = 0$

$f(x) = -x^{-1}$ $\quad f'(1)^+ = 1x^{-2} = \frac{1}{x^2} = \frac{1}{(1)^2} = 1$

$f'(1)^- \neq f'(1)^+$ $\quad$ $\boxed{\text{not differentiable}}$

14 Definitions of Deriv.

$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f(x)=3x^2-4x+1$$

$$\lim_{x\to a}\frac{3x^2-4x+\cancel{1}-(3a^2-4a+\cancel{1})}{x-a}$$

$$\lim_{x\to a}\frac{3x^2-3a^2-4x+4a}{x-a}$$

$$\lim_{x\to a}\frac{3\cancel{(x-a)}(x+a)-4\cancel{(x-a)}}{\cancel{x-a}}$$

$$=3(a+a)-4=\boxed{6a-4}$$

$x^2+2xh+h^2$

$$\lim_{h\to 0}\frac{3(x+h)^2-4(x+h)+1-(3x^2-4x+1)}{h}$$

$$\lim_{h\to 0}\frac{\cancel{3x^2}+6xh+3h^2-\cancel{4x}+4h+\cancel{1}-\cancel{3x^2}+\cancel{4x}\cancel{-1}}{h}$$

$$\lim_{h\to 0}\frac{\cancel{h}(6x+3h+4)}{\cancel{h}}$$

$$=6x+4$$

DERIVATIVES

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\tan^{-1} x = \frac{1}{x^2+1}$$

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$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}a^x = \ln a \cdot a^x$$

$$f(x) = \left(\frac{\ln\left(6e^{4x}+1\right)^3}{8^{\csc^{-1}(x^7)}}\right)\cdot \sec^8\left(7\sin^4 x + x^3\right)$$

$$f'(x) = \underbrace{\left(\frac{\ln\left(6e^{4x}+1\right)^3}{8^{\csc^{-1}(x^7)}}\right)}_{\text{1st}} \cdot \overbrace{8\sec^7\left(7\sin^4 x + x^3\right)\cdot \sec\left(7\sin^4 x + x^3\right)\tan\left(7\sin^4 x + x^3\right)\cdot\left[28\sin^3 x\cdot\cos x + 3x^2\right]}^{d'\text{2nd}}$$

$$+ \sec^8\left(7\sin^4 x + x^3\right)\cdot\left[\frac{8^{\csc^{-1}(x^7)}\cdot\frac{1}{\left(6e^{4x}+1\right)^3}\cdot 3\left(6e^{4x}+1\right)^2\cdot 6e^{4x}\cdot 4 - \ln\left(6e^{4x}+1\right)^3\cdot\ln 8\cdot 8^{\csc^{-1}(x^7)}\cdot\frac{-1}{|x^7|\sqrt{x^{14}-1}}\cdot 7x^6}{\left[8^{\csc^{-1}(x^7)}\right]^2}\right]$$

$$f(x) = x^{\cos x^2}$$

$$f(x) = e^{\ln x^{\cos x^2}}$$

$$= e^{\cos(x^2) \cdot \ln x}$$

$$f'(x) = e^{\cos(x^2) \cdot \ln x} \cdot \left[\cos(x^2) \cdot \frac{1}{x} + \ln x \cdot -\sin(x^2) \cdot 2x\right]$$

$$= x^{\cos x^2}\left[\frac{\cos x^2}{x} - \frac{2x^2 \sin(x^2) \ln x}{x}\right]$$

$$= \frac{x^{\cos x^2}}{x^1}\left[\cos x^2 - 2x^2 \sin(x^2) \ln x\right]$$

$$= x^{\cos(x^2) - 1}\left[\cos x^2 - 2x^2 \sin(x^2) \ln x\right]$$

$$=$$