December 11, 2024

Calculus Semester Review - Day 2

#26 Implicit Differentiation

Find $\frac{dy}{dx}$.

$4x^3 \sec y^2 + 8^{y^5} = 9 - \tan^{-1}(x^3)$ $\qquad \frac{1}{x^6+1}$

$4x^3 \cdot \sec y^2 \tan y^2 \cdot 2y \cdot \frac{dy}{dx} + \sec y^2 \cdot 12x^2 + \ln 8 \cdot 8^{y^5} \cdot 5y^4 \frac{dy}{dx} = \frac{-1}{x^6+1} \cdot 3x^2$

$8x^3 y \sec y^2 \tan y^2 \frac{dy}{dx} + 12x^2 \sec y^2 + 5 \ln 8 \cdot y^4 \cdot 8^{y^5} \frac{dy}{dx} = \frac{-3x^2}{x^6+1}$

$\frac{dy}{dx}\left[8x^3 y \sec y^2 \tan y^2 + 5 \ln 8 \cdot y^4 \cdot 8^{y^5}\right] = -\frac{3x^2}{x^6+1} - \frac{12x^2 \sec y^2 (x^6+1)}{1\,(x^6+1)}$

$\frac{dy}{dx}\left[8x^3 y \sec y^2 \tan y^2 + 5 \ln 8 \cdot y^4 \cdot 8^{y^5}\right] = \frac{-3x^2 - 12x^6 \sec y^2 - 12x^2 \sec y^2}{(x^6+1)}$

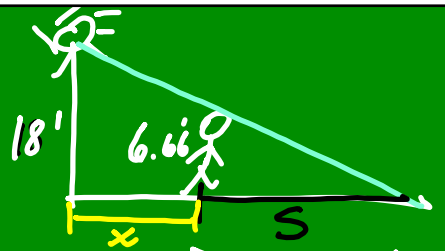
$$\frac{dy}{dx} = \frac{-3x^2 - 12x^6 \sec y^2 - 12x^2 \sec y^2}{(x^6+1)\left[8x^3 y \sec y^2 \tan y^2 + 5 \ln 8 \cdot y^4 \cdot 8^{y^5}\right]}$$

$(0,1)$ → x's $= 0$, y's $= 1$ $\quad m = \_\_\_\_$

$y - 1 = m(x - 0)$

16) $m = f'(1)$

Lilli 27



$10 \frac{ft}{s}$

How fast is shadow lengthening? When 40 ft from post

$$\frac{18}{x+s} = \frac{6.66}{s}$$

$$18s = 6.66x + 6.66s$$

$$-6.66s$$

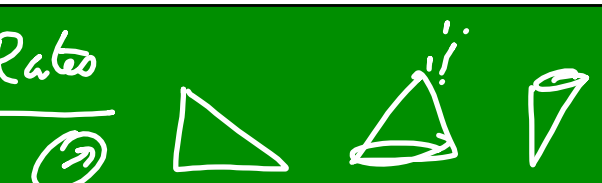
$$\frac{d}{dt}\left[11.34s = 6.66x\right]$$

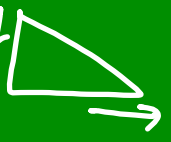
$$11.33 \frac{ds}{dt} = 6.66 \frac{dx}{dt}$$

$$11.33 \frac{ds}{dt} = 6.66 \cdot 10$$

$$\frac{ds}{dt} = \frac{66.6}{11.33} = 5.88 \text{ ft/s}$$

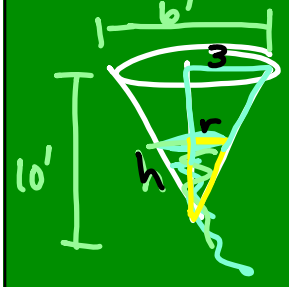
Related Rates

Geometry = 

Pythag Thm: 

Angle of elev. / = trig func. — opp, adj, hyp

2 parts. 

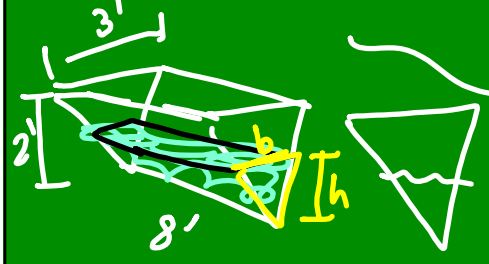


$V = \frac{1}{3}\pi r^2 h$ Find rate height is changing?

$\frac{3}{10} = \frac{r}{h}$ $V = \frac{1}{3}\pi(0.3h)^2 h$

$\frac{3}{10}h = r$ $\frac{d}{dt}\left[V = 0.03\pi h^3\right]$

$\frac{dV}{dt} = 0.09\pi h^2 \frac{dh}{dt}$



$V = \frac{1}{2}bh\ell$ $\ell = 8$

$V = 4bh$

$\frac{3}{2} = \frac{b}{h}$