

CURVE SKETCHING

2/ Inc/Dec

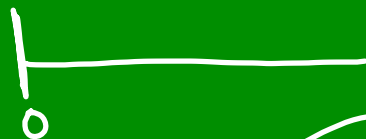
$$f'(x) = 0$$

Find crit pts.



Test pts in f'

2(b) $\ln x$ Domain $(0, \infty)$



Points of non-differentiability

$$f(x) = 6x^{2/3} - 12x$$

$$f'(x) = 4x^{-1/3} - 12 = 0$$

$$\frac{4}{\sqrt[3]{x}} - 12 = 0$$

$$\begin{aligned} \frac{4}{\sqrt[3]{x}} &= 12 \\ \left(\frac{1}{3}\right)^3 &= (\sqrt[3]{x})^3 \end{aligned}$$

$$x = 0 \quad \frac{1}{27} = x$$

3) Concavity

$$f''(x) = 0$$

Test pts on f''

Relative Extrema

all peaks/valleys

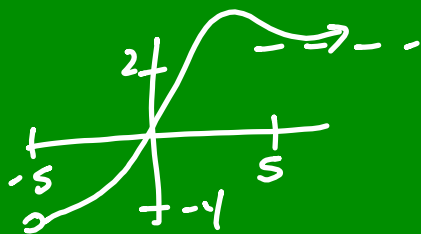
First Deriv. Test

- 1) Find crit pts.
- 2) Test Pts.
- 3) Mountain Test

Second Deriv. Test

- 1) Find crit pts.
- 2) Sub in crit pts in f''

$$f''(1/2) = 8 + \text{✗}$$

 $(-5, 5)$

$$\lim_{x \rightarrow -5} = -4$$

$$\lim = 2$$

Absolute Extrema $(-\infty, \infty)$

Parentheses: Limits

$$[-3, 8]$$

-3	
8	
crit pts	

$$d) f(x) = \frac{e^x}{x} \quad (0, \infty)$$

$$\lim_{x \rightarrow 0^+} \frac{e^x}{x} = \frac{1}{0} = \frac{+}{+} = +\infty$$

$$\lim_{x \rightarrow \infty} \frac{e^x}{x} = \frac{\infty}{\infty}$$

$$\lim_{x \rightarrow \infty} \frac{e^x}{1} = \frac{\infty}{1} = \infty$$

No abs max

$$f'(x) = \frac{x \cdot e^x - e^x \cdot 1}{x^2}$$

$$0 = \frac{e^x(x-1)}{x^2}$$

$$x-1=0$$

$$x=1$$

$$\frac{1}{1} \frac{e^1}{1} = e$$

Abs min

 $(1, e)$

No Abs max

AsymptotesVertical

Denom = 0

$$\lim_{x \rightarrow \#} f(x) = \pm \infty$$

Horiz

$$\lim_{x \rightarrow \infty} f(x) = \#$$

Slant/Curvilinear

↑
num.
is
1 power
higher

↑
num is
2+ powers
higher

$$f(x) = \frac{4x^4 - 8x^2 + 2x + 1}{2x^2 - 2x + 4}$$

$$2x^2 + 2x - 6$$

$$\begin{array}{r} 2x^2 - 2x + 4 \overline{) 4x^4 + 0x^3 - 8x^2 + 2x + 1} \\ \underline{-4x^4 + 4x^3 + 8x^2} \\ 4x^3 - 16x^2 + 2x \\ \underline{-4x^3 + 4x^2 + 8x} \\ -12x^2 - 6x + 1 \end{array}$$

$$y = 2x^2 + 2x - 6$$

Vertex $x = -\frac{b}{2a} = -\frac{2}{4} = -\frac{1}{2}$

$$y = 2\left(-\frac{1}{2}\right)^2 + 2\left(-\frac{1}{2}\right) - 6$$

$$y = -6.5$$

$$\left(-\frac{1}{2}, -6.5\right)$$

$$\begin{array}{r|l} 0 & 6 \\ 2 & 2 \\ 3 & 18 \end{array}$$

9 Graph of Derivative

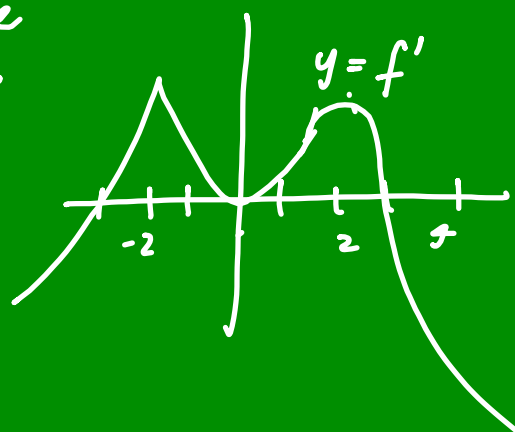
Crit Pts

-3, 0, 3

$\frac{-}{-3} \frac{+}{0} \frac{+}{3} \frac{-}{}$

(Above/
Below
x-axis)

$\swarrow -3 \searrow 0 \swarrow 3 \searrow$



Concavity

Possible Inf pts (Peaks/Valleys)

$\frac{+}{-2} \frac{-}{0} \frac{+}{2} \frac{-}{}$

← graph goes
up/down

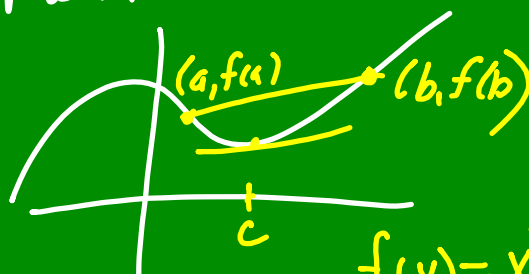
Concave up $(-\infty, -2) (0, 2)$

down $(-2, 0) (2, \infty)$

CAS - Partial Problem

Find asymptotes
of inc/dec
or
concavity

Mean Value Thm



$$f'(c) = \frac{f(b) - f(a)}{b - a} \quad [-1, 3]$$

$$2x + 2 = \frac{f(3) - f(-1)}{3 - (-1)}$$

$$2c + 2 = \frac{15 + 1}{4} = \frac{16}{4}$$

$$2c + 2 = 4$$

$$2c = 2$$

$$c = 1$$