Antidifferentiation (Integration)

$y = f(x)$

$\frac{dy}{dx} = f'(x)$

$\int dy = \int f'(x)\,dx$

$y = \int f'(x)\,dx$

$\int (4x^1 + 9x^2)\,dx$

$\frac{4x^2}{2} + \frac{9x^3}{3}$

$\boxed{2x^2 + 3x^3 + C}$

Power Rule for Intg

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

Indefinite Integrals

expression + C

Definite integrals

= numerical value

$$\int \left(\frac{2}{x^3} + 4\sqrt[3]{x} - 2x^{3/5} + 7 \right) dx$$

$$\int \left(2x^{-3} + 4x^{1/3} - 2x^{3/5} + 7 \right) dx \qquad 7x^0$$

$$= \frac{2x^{-2}}{-2} + \frac{3}{4}\cdot 4x^{4/3} - \frac{5}{8}\cdot 2x^{8/5} + 7x + C$$

$$= \frac{-1}{x^2} + 3x^{4/3} - \frac{5}{4}x^{8/5} + 7x + C$$

$$\int (7x-4)^2 \, dx \qquad (7x-4)(7x-4)$$

$$\int (49x^2 - 56x + 16) \, dx$$

$$= \frac{49x^3}{3} - \frac{56x^2}{2} + 16x + C$$

$$= \frac{49}{3}x^3 - 28x^2 + 16x + C$$

$$\int \frac{4x^2-2x+1}{\sqrt{x}}\,dx$$

$$\int (4x^2-2x+1)\cdot x^{-1/2}\,dx$$

$$\int \left(4x^{3/2}-2x^{1/2}+x^{-1/2}\right)dx$$

$$= \frac{2}{5}\cdot 4x^{5/2}-\frac{2}{3}\cdot 2x^{3/2}+2\cdot x^{1/2}+C$$

$$= \frac{8}{5}x^{5/2}-\frac{4}{3}x^{3/2}+2x^{1/2}+C$$

Initial value problem.

Find y.

$$\int \frac{dy}{dx} = \int (3x^2+2x)\,dx \qquad y(2)=7$$

(x = 2, y = 7)

$$y = \frac{3x^3}{3}+\frac{2x^2}{2}+C$$

$$y = x^3+x^2+C$$

$$7 = (2)^3+(2)^2+C$$

$$7 = 8+4+C$$

$$-5 = C$$

$$\boxed{y = x^3+x^2-5}$$

U-Substitution

$$\int 6x\,(x^2+5)^8\,dx$$

$$\int 6x \cdot u^8\,dx$$

$$\int 6x \cdot u^8 \cdot \frac{du}{2x}$$

$$3\int u^8\,du$$

$$3 \cdot \frac{u^9}{9} + C$$

$$\frac{1}{3}(x^2+5)^9 + C$$

$u = x^2 + 5$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\frac{du}{2x} = dx$

$$\int \frac{3x}{(4-3x^2)^7}\,dx$$

$$\int \frac{3x}{u^7} \cdot \frac{du}{-6x}$$

$$-\frac{1}{2}\int u^{-7}\,du$$

$$-\frac{1}{2} \cdot \frac{u^{-6}}{-6} + C$$

$$\frac{1}{12u^6} + C$$

$$\frac{1}{12(4-3x^2)^6} + C$$

$u = 4 - 3x^2$

$du = -6x\,dx$

$\frac{du}{-6x} = dx$

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$$\frac{d}{dx}\sin x = \cos x \qquad \int \cos x\,dx = \sin x + C$$

$$\frac{d}{dx}\cos x = -\sin x \qquad \int \sin x\,dx = -\cos x + C$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \int \sec^2 x\,dx = \tan x + C$$

$$\frac{d}{dx}\cot x = -\csc^2 x \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\frac{d}{dx}\sec x = \sec x \tan x \qquad \int \sec x \tan x\,dx = \sec x + C$$

$$\frac{d}{dx}\csc x = -\csc x \cot x \qquad \int \csc x \cot x\,dx = -\csc x + C$$

$$\frac{d}{dx}e^x = e^x \qquad \int e^x\,dx = e^x + C$$

$$\frac{d}{dx}\ln x = \frac{1}{x} + C \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int \left(4\csc x \cot x + 3\cos x - \frac{2}{x}\right) dx$$

$$-2 \cdot \frac{1}{x}$$

$$= -4\csc x + 3\sin x - 2\ln|x| + C$$