More Integration

$$\int 5x\sqrt{2x+3}\,dx$$

$$\int 5x \cdot u^{1/2} \cdot \frac{du}{2}$$

$$\frac{5}{2}\int \frac{u-3}{2} \cdot u^{1/2}\,du$$

$$\frac{5}{4}\int \left(u^{3/2} - 3u^{1/2}\right) du$$

$$\frac{5}{4}\left[\frac{2}{5}u^{5/2} - \frac{3\cdot 2}{1}u^{3/2}\right] + C$$

$$\frac{1}{2}u^{5/2} - \frac{5}{2}u^{3/2} + C$$

$$\frac{1}{2}(2x+3)^{5/2} - \frac{5}{2}(2x+3)^{3/2} + C$$

$u = 2x+3$

$du = 2\,dx$

$\frac{du}{2} = dx$

$\frac{u-3}{2} = \frac{2x}{2}$

$$\int (x+2)^2\sqrt{1+x}\,dx$$

$u = 1+x \quad u-1 = x$

$du = 1\,dx$

$$\int (x+2)^2 \cdot u^{1/2}\,du$$

$$\int (u-1+2)^2 \cdot u^{1/2}\,du$$

$$\int (u+1)^2 \cdot u^{1/2}\,du$$

FOIL!

$$\int (u^2+2u+1)u^{1/2}\,du$$

$$\int \left(u^{5/2}+2u^{3/2}+u^{1/2}\right)du$$

$$= \frac{2}{7}u^{7/2} + 2\cdot\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{7}(1+x)^{7/2} + \frac{4}{5}(1+x)^{5/2} + \frac{2}{3}(1+x)^{3/2} + C$$

$$\int \tan^8 x \sec^2 x \, dx$$

$$\int u^8 \sec^2 x \cdot \frac{du}{\sec^2 x}$$

$$= \frac{u^9}{9} + C$$

$$= \frac{\tan^9 x}{9} + C$$

$$u = \tan x$$

$$du = \sec^2 x \, dx$$

$$\frac{du}{\sec^2 x} = dx$$

$$\int x^6 \sin(x^7)\,dx$$

$$u = x^7$$
$$du = 7x^6\,dx$$
$$\frac{du}{7x^6} = dx$$

$$\int x^6 \cdot \sin u \cdot \frac{du}{7x^6}$$

$$\frac{1}{7}\int \sin u\,du$$

$$= -\frac{1}{7}\cos u + C$$

$$= \boxed{-\frac{1}{7}\cos(x^7) + C}$$

$$\int \frac{1}{y^2} \sec\left(\frac{1}{y}\right) \tan\left(\frac{1}{y}\right) dy$$

$$u = \frac{1}{y} = y^{-1}$$

$$du = -1y^{-2} dy$$

$$du = -\frac{1}{y^2} dy$$

$$-y^2 du = dy$$

$$\int \frac{1}{y^2} \sec u \tan u \cdot -y^2 du$$

$$= -\sec u + C$$

$$= -\sec\left(\frac{1}{y}\right) + C$$

$$\int \frac{8\cos(4x-7)}{\sin^6(4x-7)}\,dx$$

$u = \sin(4x-7)$

$du = \cos(4x-7)\cdot 4\,dx$

$\frac{du}{4\cos(4x-7)} = dx$

$$\int \frac{\overset{2}{\cancel{8}}\cancel{\cos(4x-7)}}{u^6}\cdot\frac{du}{\cancel{4}\cancel{\cos(4x-7)}}$$

$$2\int u^{-6}\,du$$

$$2\,\frac{u^{-5}}{-5}+C$$

$$\boxed{-\frac{2}{5(\sin^5(4x-7))}+C}$$