Integration of Exponential, Logarithmic, & Inv. Trig Functions

$\frac{d}{dx} e^x = e^x$ $\qquad \int e^x \, dx = e^x + C$

$\frac{d}{dx} a^x = \ln a \cdot a^x$ $\qquad \int a^x \, dx = \frac{1}{\ln a} a^x + C$

$\int 8^x \, dx = \frac{1}{\ln 8} \cdot 8^x + C$

$\int x \cdot e^{5x^2} dx$ $\qquad u = 5x^2$

$du = 10x \, dx$

$\frac{du}{10x} = dx$

$\int \cancel{x} \cdot e^u \cdot \frac{du}{10\cancel{x}}$

$\frac{1}{10} \int e^u \, du$

$\frac{1}{10} e^u + C$

$\boxed{\frac{1}{10} e^{5x^2} + C}$

$\int \frac{e^{\tan y}}{\cos^2 y} dy$ $\qquad u = \tan y$

$du = \sec^2 y \, dy$

$\frac{du}{\sec^2 y} = dy$

$\int \frac{e^u}{\cos^2 y} \frac{du}{\sec^2 y}$ $\quad \frac{1}{\cos^2 y}$

$\int \frac{e^u}{\cancel{\cos^2 y}} \cdot \cancel{\cos^2 y} \, du$

$= e^u + C$

$= e^{\tan y} + C$

$$\int 3x^5 \cdot 7^{x^6}\,dx$$

$u = x^6$

$du = 6x^5\,dx$

$$\int \cancel{3x^5} \cdot 7^u \frac{du}{\cancel{6x^5}}$$

$$\frac{1}{2}\int 7^u\,du$$

$$\frac{1}{2} \cdot \frac{1}{\ln 7} \cdot 7^u + C$$

$$\boxed{\frac{1}{2\ln 7} \cdot 7^{x^6} + C}$$

$$\frac{d}{dx}\ln x = \frac{1}{x} \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int \frac{x^2}{1-x^3}\,dx \qquad u = 1-x^3$$
$$du = -3x^2\,dx$$

$$\int \frac{\cancel{x^2}}{u}\,\frac{du}{-3\cancel{x^2}}$$

$$-\frac{1}{3}\int \frac{1}{u}\,du$$

$$-\frac{1}{3}\ln|u| + C$$

$$-\frac{1}{3}\ln|1-x^3| + C$$

$$\int \frac{\csc^2 x}{\cot x}\,dx \qquad u = \cot x$$
$$du = -\csc^2 x\,dx$$

$$\int \frac{\cancel{\csc^2 x}}{u}\,\frac{du}{-\cancel{\csc^2 x}}$$

$$-\int \frac{1}{u}\,du$$

$$-\ln|u| + C$$

$$-\ln|\cot x| + C$$

$$\int \frac{(\ln z)^5}{z}\,dz \qquad u = \ln z$$
$$du = \frac{1}{z}\,dz$$
$$z\,du = dz$$

$$\int \frac{u^5}{\cancel{z}}\,\cancel{z}\,du$$

$$= \frac{u^6}{6} + C$$

$$= \frac{(\ln z)^6}{6} + C$$

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}} \qquad \int\frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{x^2+1} \qquad \int\frac{1}{x^2+1}\,dx = \tan^{-1}x + C$$

$$\frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}} \qquad \int\frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1}x + C$$

$$\int\frac{6x^2}{\sqrt{1-25x^6}}\,dx$$

$$6\int\frac{x^2}{\sqrt{1-(5x^3)^2}}\,dx \qquad \begin{aligned} u &= 5x^3 \\ du &= 15x^2\,dx \end{aligned}$$

$$= 6\int\frac{\cancel{x^2}}{\sqrt{1-u^2}}\cdot\frac{du}{15\cancel{x^2}}$$

$$= \frac{2}{5}\int\frac{1}{\sqrt{1-u^2}}\,du$$

$$= \frac{2}{5}\sin^{-1}u + C$$

$$= \frac{2}{5}\sin^{-1}(5x^3) + C$$

$$\int \frac{3x}{4+9x^4}\,dx$$

$$\frac{3}{4}\int \frac{x}{1+\frac{9}{4}x^4}\,dx$$

$$\frac{3}{4}\int \frac{x}{1+\left(\frac{3}{2}x^2\right)^2}\,dx \qquad u=\frac{3}{2}x^2 \qquad du = 3x\,dx$$

$$\frac{\cancel{3}}{4}\int \frac{\cancel{x}}{1+u^2}\,\frac{du}{\cancel{3}\cancel{x}}$$

$$\frac{1}{4}\int \frac{1}{1+u^2}\,du$$

$$\frac{1}{4}\tan^{-1}u + C$$

$$\frac{1}{4}\tan^{-1}\left(\frac{3}{2}x^2\right) + C$$

1) Make the "1"

2) Make the squared quantity $(\quad)^2$

3) u-sub the squared quantity

$$\int \frac{4\cos x}{\sin x\sqrt{\sin^2 x-36}}\,dx$$

$$\frac{4}{6}\int \frac{\cos x}{\sin x\sqrt{\frac{\sin^2 x}{36}-1}}\,dx$$

$$\frac{2}{3}\int \frac{\cos x}{\sin x\sqrt{\left(\frac{\sin x}{6}\right)^2-1}}\,dx \qquad u=\frac{\sin x}{6}=\frac{1}{6}\sin x$$

$$du=\frac{1}{6}\cos x\,dx$$

$$\frac{2}{3}\int \frac{\cancel{\cos x}}{\cancel{6}u\sqrt{u^2-1}}\cdot\frac{\cancel{6}\cdot du}{\cancel{\cos x}} \qquad u=\frac{\sin x}{6}$$

$$6u=\sin x$$

$$\frac{2}{3}\int \frac{1}{u\sqrt{u^2-1}}\,du$$

$$\frac{2}{3}\sec^{-1}u+C$$

$$\frac{2}{3}\sec^{-1}\left(\frac{\sin x}{6}\right)+C$$