

PHASE SHIFT & VERTICAL SHIFT

$$y = x^2$$

$$y = (x-5)^2 + 2$$

↑ Right ↑ Up 2

$$y = \sin\left(x + \frac{\pi}{4}\right) - 3$$

Left $\frac{\pi}{4}$ ↑ Down 3

$$y = \sin\left(2\left(x + \frac{\pi}{4}\right)\right) - 3$$

$$y = \sin\left(2x + \frac{\pi}{2}\right) - 3$$

$$2x + \frac{\pi}{2} = 0$$

$$\frac{1}{2} \cdot 2x = -\frac{\pi}{2} \cdot \frac{1}{2}$$

$$x = -\frac{\pi}{4}$$

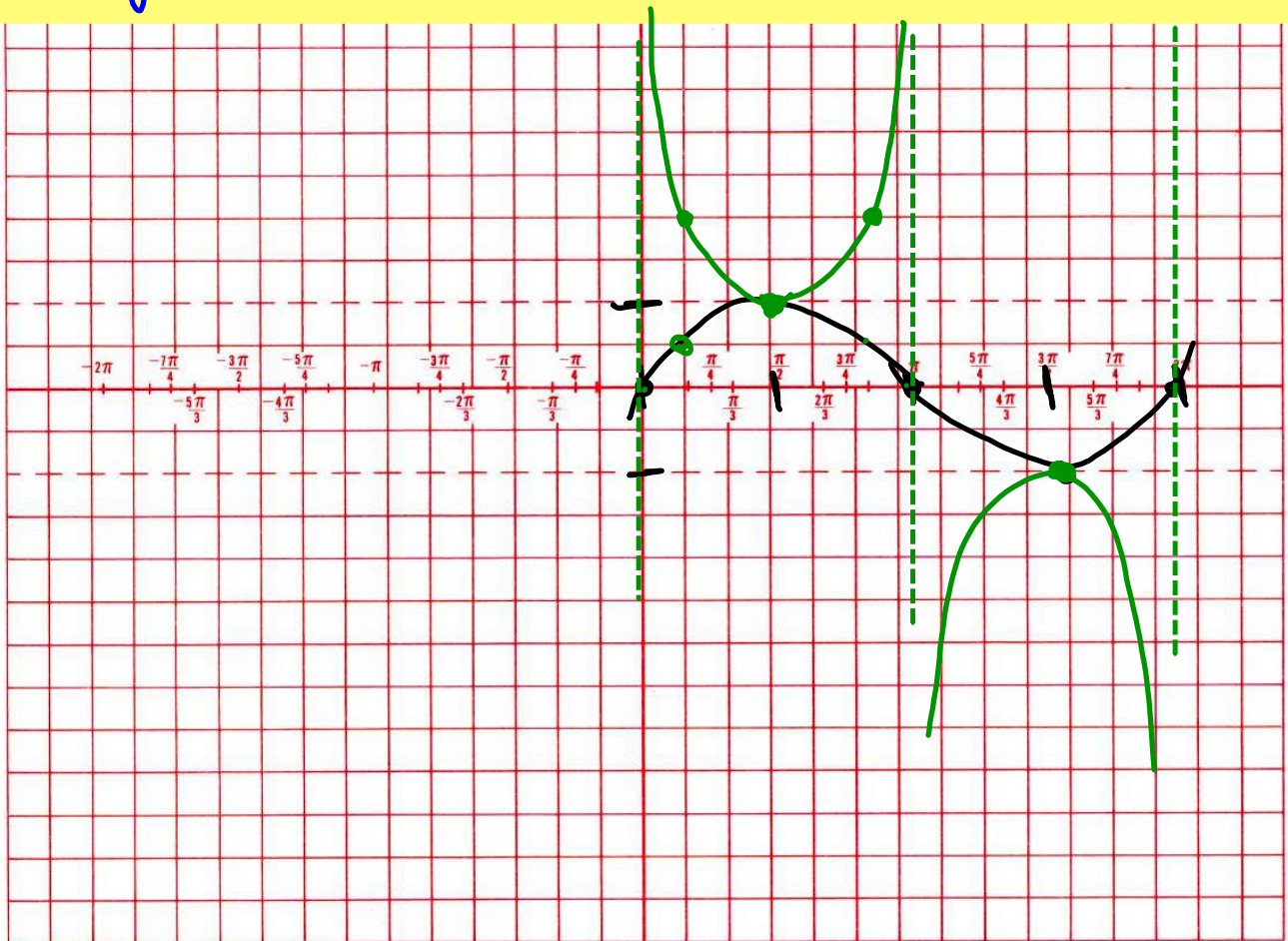
With a little $\sec x$ & $\csc x$

$$y = a \frac{\sin}{\cos} (bx+c) + d$$

	$\frac{\sin}{\cos}$	$\frac{\csc}{\sec}$
amp	$ a $	NA
period	$\frac{2\pi}{b}$	$\frac{2\pi}{b}$
vertical shift	d	d
phase shift (horiz shift)	$bx+c=0$ $x = -\frac{c}{b}$	$bx+c=0$

$$y = \sin x \quad \frac{0}{1} \quad \frac{1}{0}$$

$$y = \csc x$$

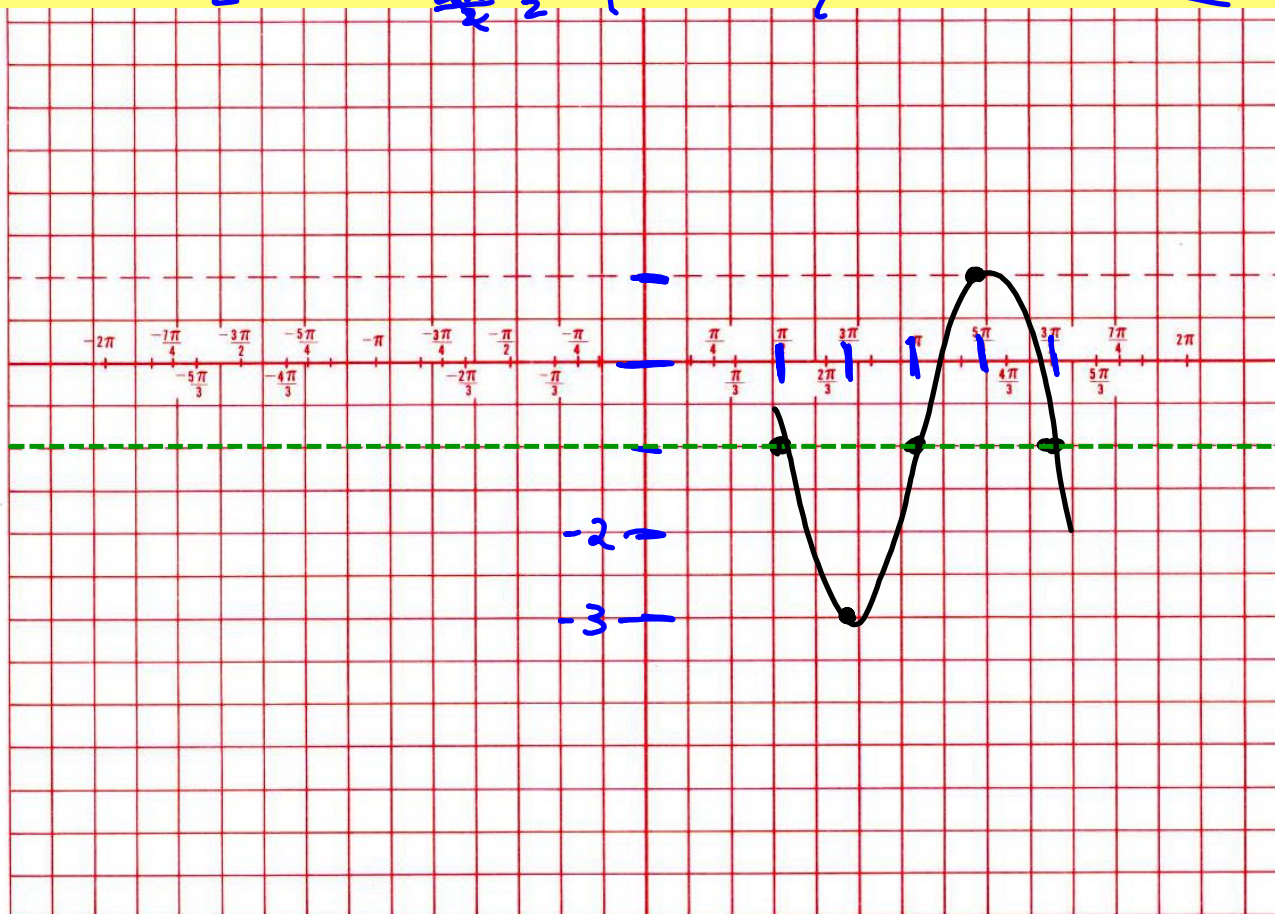


$$y = -2 \sin(2x - \pi) - 1$$

amp $\frac{2(-)}{2} = -1$ per $\frac{2\pi}{2} = \pi$ v.s. $\frac{p.s.}{2x - \pi = 0} \Rightarrow 2x = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{4}$

Spacing $\frac{\pi}{4} = \frac{\pi}{4}$

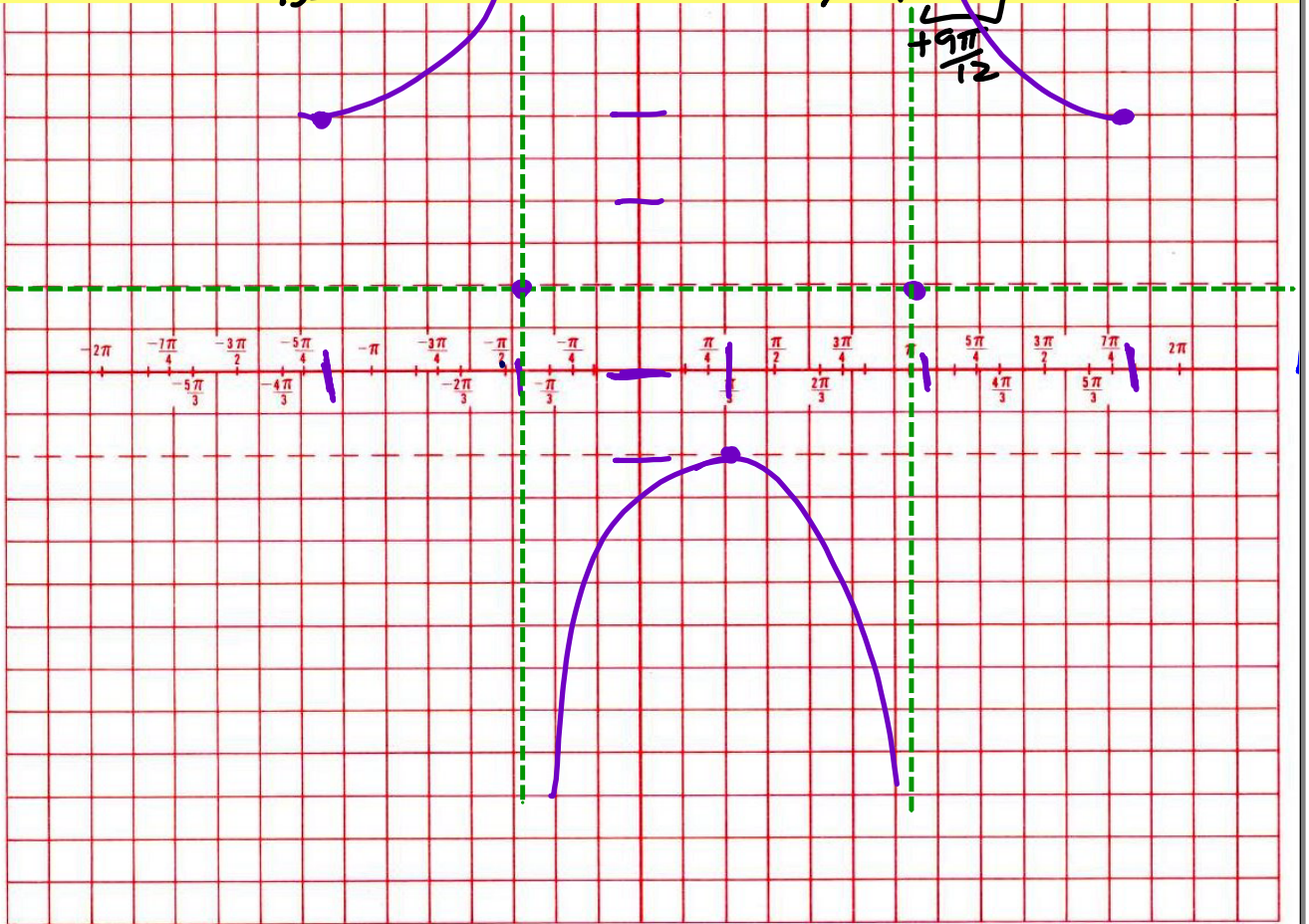
$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$
$\frac{2\pi}{4}$	$\frac{3\pi}{4}$	$\frac{4\pi}{4}$	$\frac{5\pi}{4}$	$\frac{6\pi}{4}$

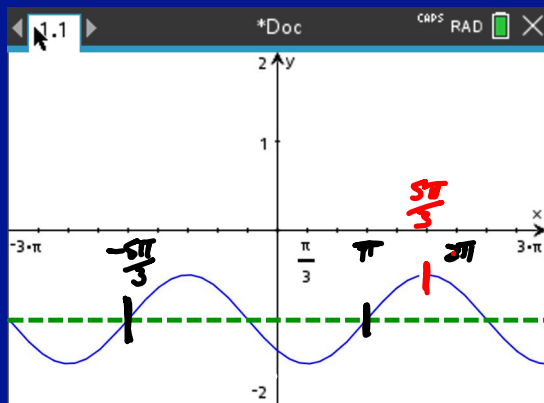


$$y = 1 + 2 \sec\left(\frac{2}{3}x + \frac{7\pi}{9}\right)$$

Amp $NA(2)$ per $\frac{2\pi \cdot 3}{\frac{2}{3}} = 3\pi$ V.S. 1 P.S. $x = -\frac{7\pi}{9} \cdot \frac{3}{2} = -\frac{7\pi}{6}$ Spacing $\frac{3\pi \cdot \frac{1}{3}}{4} = \frac{3\pi}{4}$

$-\frac{7\pi}{6}$	$-\frac{5\pi}{12}$	$\frac{\pi}{3}$	$\frac{13\pi}{12}$	$\frac{11\pi}{6}$
$-\frac{11\pi}{12}$	$-\frac{5\pi}{12}$	$\frac{4\pi}{12}$	$\frac{13\pi}{12}$	$\frac{22\pi}{12}$





COS X

$$V.S = -1$$

$$\text{amp} = \frac{1}{2}$$

$$\text{p.s.} = \frac{5\pi}{3}$$

$$\text{horiz}$$

$$\text{period} = \frac{2\pi}{b}$$

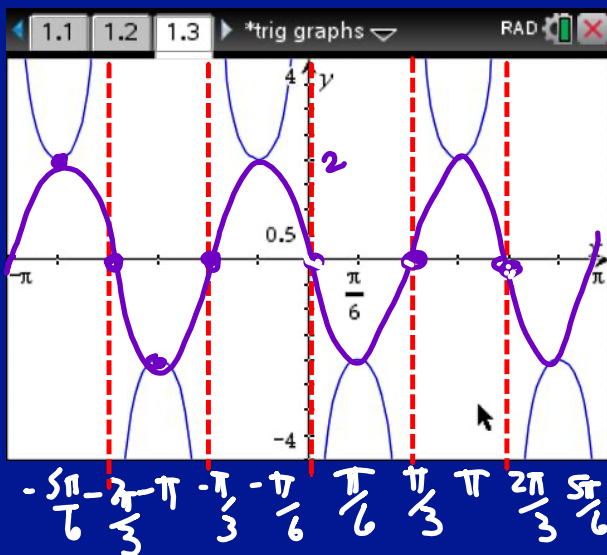
$$\pi + \frac{5\pi}{3} = \frac{8\pi}{3}$$

$$b = \frac{2\pi}{\text{per}}$$

$$= \frac{2\pi}{\frac{8\pi}{3}}$$

$$= 2\pi \cdot \frac{3}{8\pi}$$

$$y = \frac{1}{2} \cos \left[\frac{3}{4} \left(x - \frac{5\pi}{3} \right) \right] - 1$$



CSC X

$$V.S. = 0$$

$$\text{amp.} = 2$$

$$\text{period} = \frac{2\pi}{3} \quad b = \frac{2\pi}{\frac{2\pi}{3}} = 3$$

$$\text{p.s.} = \frac{\pi}{3} \quad \left(-\frac{\pi}{3}, \pi \right)$$

$$y = 2 \csc \left(3 \left(x - \frac{\pi}{3} \right) \right)$$

$$y = -2 \csc(3x)$$

$$y = -2 \csc \left(3 \left(x + \frac{\pi}{3} \right) \right)$$

$$(x - \pi)$$

$$(x + \pi)$$