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Rational Exponents

Rule #6

$$a^{m/n} = \sqrt[n]{a^m}$$

$$x^{2/3} = \sqrt[3]{x^2}$$

$$\sqrt[4]{p^3} = p^{3/4}$$

$$\sqrt[7\cdot2]{x^{2\cdot2}y^{3\cdot2}} \cdot \sqrt[7]{x^7y^7}$$

$$\sqrt[14]{x^4y^6} \cdot \sqrt[14]{x^7y^7}$$

$$\boxed{\sqrt[14]{x^{11}y^{13}}}$$

Rational #'s =

$$\sqrt[4\cdot3]{a^{3\cdot3}b^{2\cdot3}} \cdot \sqrt[3\cdot4]{a^4b^{2\cdot4}}$$

$$= a^{\frac{3}{4}}b^{\frac{1}{4}} \cdot a^{\frac{1}{3}}b^{\frac{2}{3}}$$

$$= a^{\frac{9}{12}}b^{\frac{3}{12}} \cdot a^{\frac{4}{12}}b^{\frac{8}{12}}$$

$$= a^{\frac{13}{12}}b^{\frac{11}{12}}$$

$$= \sqrt[12]{a^{13}b^{11}}$$

$$= a\sqrt[12]{a^1b^{11}}$$

Shortcut

$$\sqrt[12]{a^9b^3} \cdot \sqrt[12]{a^4b^8}$$

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Evaluate.

$8^{1/3} = \sqrt[3]{8^1} = 2$

$25^{3/2} = \sqrt[2]{25^3} = 5^3 = 125$

$16^{3/4} = \sqrt[4]{16^3} = 2^3 = 8$

$81^{-1/2} = \frac{1}{\sqrt{81^1}} = \frac{1}{9}$

$32^{-2/5} = \frac{1}{\sqrt[5]{32^2}} = \frac{1}{2^2} = \frac{1}{4}$

$\left(\frac{49}{16}\right)^{-3/2} = \left(\frac{16}{49}\right)^{3/2} = \sqrt[2]{\left(\frac{16}{49}\right)^3} = \left(\frac{4}{7}\right)^3 = \boxed{\frac{64}{343}}$

Write as a single radical & simplify.

$\sqrt[5]{\sqrt[3]{x^1}} = \sqrt[15]{x}$

$(x^{1/3})^{1/5}$

$\sqrt[n]{\sqrt[m]{a}} = \sqrt[m \cdot n]{a}$

$\sqrt[4]{\sqrt[3]{\sqrt[2]{x}}} = \sqrt[24]{x}$

SOLVE.

Quadratic Form.

$x^2 - 2x^1 - 3 = 0$

$(x^1 + 1)(x^1 - 3) = 0$

$x + 1 = 0 \quad x - 3 = 0$

$x = -1 \quad x = 3$

$\left(x^{2/3}\right)^{3/2} = (4)^{3/2}$

Which of these is in quadratic form?

$x^6 - 2x^3 - 8 = 0$ yes

$x^{2/5} - 3x^{1/5} + 2 = 0$ yes

$x^8 + 3x^2 + 2 = 0$ No

* Power on middle term is 1/2 of Power on 1st term
* Factor using the power on the middle term.

Solve for x.

$x^{2/3} - 3x^{1/3} - 28 = 0$

1 28
2 14
4 7

$(x^{1/3} + 4)(x^{1/3} - 7) = 0$

$x^{1/3} + 4 = 0 \quad x^{1/3} - 7 = 0$

$(x^{1/3})^3 = (-4)^3 \quad (x^{1/3})^3 = (7)^3$

$x = -64 \quad x = 343$

Solving Radical Equations

$5\sqrt[3]{x+7} - 10 = 5$
$+10 \quad +10$

$\frac{5\sqrt[3]{x+7}}{5} = \frac{15}{5}$

$\left(\sqrt[3]{x+7}\right)^3 = (3)^3$

$x+7 = 27$
$-7 \quad -7$

$x = 20$

$1 + 2 = 3$

1) Isolate the root
2) Raise both sides to a power to cancel the root.

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ULTIMATE PROBLEM

$\sqrt{2x-2} - \sqrt{3x-2} = -1$

$\left(\sqrt{2x-2}\right)^2 = \left(\sqrt{3x-2} - 1\right)^2$ FOIL!

$2x-2 = \left(\sqrt{3x-2}-1\right)\left(\sqrt{3x-2}-1\right)$

$2x-2 = 3x-2-\sqrt{3x-2}-\sqrt{3x-2}+1$

$2x-2 = 3x-1-2\sqrt{3x-2}$

$-2x \quad +2$

$\left(2\sqrt{3x-2}\right)^2 = (x+1)^2$ ⟸ FOIL!

$4(3x-2) = (x+1)(x+1)$

$12x-8 = x^2+x+x+1$

$+8 \qquad -12x \quad +8$

$0 = x^2-10x+9$

$0 = (x-1)(x-9)$

$x-1=0 \quad x-9=0$

$\boxed{x=1 \quad x=9}$

Check:

$\sqrt{2x-2} - \sqrt{3x-2} = -1$

$x=1$

$\sqrt{0} - \sqrt{1} = -1$

$0-1=-1$ ✓

$x=9$

$\sqrt{16} - \sqrt{25} = -1$

$4-5=-1$ ✓

1) Isolate one root
2) Square both sides FOIL!
3) Clean up! Add like terms
4) Isolate the remaining root
5) Square both sides
6) Set =0, factor + solve
7) Check the answers!