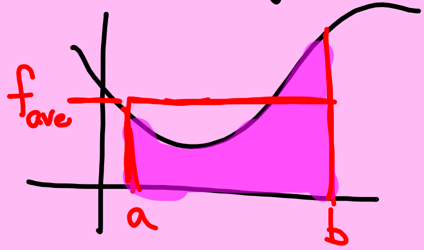
MEAN VALUE THM FOR INTEGRALS

(Average value)



$(b-a)\cdot f_{ave} = \int_a^b f(x)\,dx$

$$\boxed{f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

Area of Rect = Area under curve

formed at f_{ave}

$f(x) = x^2 - 2x + 1$

$a = 2 \quad b = 5$

$$f_{ave} = \frac{1}{5-2}\int_2^5 (x^2 - 2x + 1)\,dx$$

$$= \frac{1}{3}\left[\frac{x^3}{3} - \cancel{\frac{2x^2}{2}} + x\right]_2^5$$

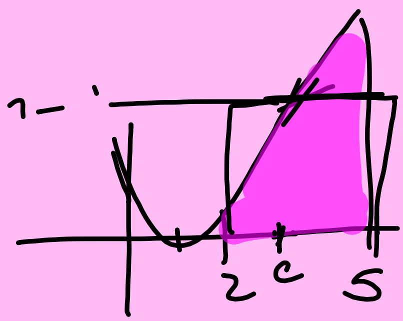
$$= \frac{1}{3}\left[\frac{125}{3} - 25 + 5 + \left(-\frac{8}{3} + 4 - 2\right)\right]$$

$$\frac{1}{3}\left[\frac{117}{3} - 18\right]$$

$$= \frac{1}{3}\left[39 - 18\right]$$

$$= \frac{1}{3}(21)$$

$$= 7$$



$x^2 - 2x + 1 = 7$

$c^2 - 2c + 1 = 7$

$c^2 - 2c - 6 = 0$

$h = f_{ave}$

$$c = \frac{2 \pm \sqrt{4 - 4(1)(-6)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{28}}{2}$$

$$= \frac{2 \pm 2\sqrt{7}}{2} = 1 \pm \sqrt{7}$$

Integration Review

$$\int \frac{2}{4x^2+9}\,dx$$

$$\frac{2}{9}\int \frac{1}{\frac{4x^2}{9}+1}\,dx \qquad \left(\tfrac{2}{3}x\right)^2$$

$$\frac{2}{9}\int \frac{1}{u^2+1}\,dx$$

$$\cancel{\frac{2}{9}}\,\frac{1}{3}\int \frac{1}{u^2+1}\,\cancel{\frac{3}{2}}\,du$$

$$\frac{1}{3}\tan^{-1}u + C$$

$$\frac{1}{3}\tan^{-1}\left(\frac{2}{3}x\right) + C$$

$$\int \frac{2x}{4x^2+9}\,dx$$

$u = 4x^2+9$

$du = 8x\,dx$

$\frac{du}{8x} = dx$

$u = \frac{2}{3}x$

$du = \frac{2}{3}\,dx$

$$\int \frac{2x+1}{4x^2}\,dx$$

$$\int \frac{\cancel{2x}}{u}\,\frac{du}{\cancel{8x}_{\,4}}$$

$$\frac{1}{4}\int \frac{1}{u}\,du$$

$$\frac{1}{4}\ln|u| + C$$

$$\frac{1}{4}\ln|4x^2+9| + C$$

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$$\int \frac{4x^5 - 2x^3}{2x^2}\,dx$$

$$\frac{1}{2}\int (4x^5 - 2x^3)\,x^{-2}\,dx$$

$$\frac{1}{2}\int (4x^3 - 2x)\,dx$$

$$\int \frac{x-2}{\sqrt{x-1}}\,dx$$

$u = x - 1$

$du = 1dx$

$u + 1 = x$

$$\int \frac{x-2}{u^{1/2}}\,du$$

$$\int \frac{u+1-2}{u^{1/2}}\,du$$

$$\int \frac{u-1}{u^{1/2}}\,du$$

$$\int (u-1)\,u^{-1/2}\,du$$

$$\int (u^{1/2} - u^{-1/2})\,du$$

$$\int x^2 \cos(4x^3)\sin^8(4x^3)\,dx$$

$$u = \sin(4x^3)$$
$$du = \cos(4x^3)\cdot 12x^2\,dx$$

$$\int \cancel{x^2}\,\cancel{\cos(4x^3)}\cdot u^8\cdot \frac{du}{12\cancel{x^2}\,\cancel{\cos(4x^3)}}$$

$$\frac{1}{12}\int u^8\,du$$

$$\int \frac{1}{x}\sec(\ln x)\tan(\ln x)\,dx$$

$$u = \ln x$$
$$du = \frac{1}{x}\,dx$$
$$x\,du = dx$$

$$\int \frac{1}{\cancel{x}}\sec u \tan u\cdot \cancel{x}\,du$$
$$= \sec u + C$$
$$= \sec(\ln x) + C$$