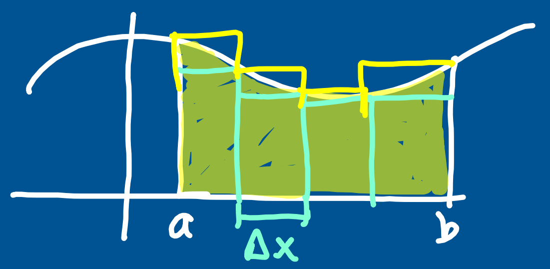
RIEMANN Sums



$f(x) = 3x^2 + 4$ $\quad [\overset{a}{1}, \overset{b}{3}]$

4 sub-intervals

$n = 4$

$$\lim_{\Delta x \to 0} \sum_{x=a}^{b} f(x) \cdot \Delta x$$

$$\text{width} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$= \frac{b-a}{n}$$

$$L = \frac{1}{2} \cdot f(1) + \frac{1}{2} f(1.5) + \frac{1}{2} f(2) + \frac{1}{2} f(2.5)$$

$$= \frac{1}{2}\left[f(1) + f(1.5) + f(2) + f(2.5)\right]$$

$$= \frac{1}{2}\left[7 + 10.75 + 16 + 22.75\right]$$

$$= \frac{1}{2}\left[56.5\right]$$

$$L = 28.25 \text{ units}^2$$

$$R = \frac{1}{2}\left[f(1.5) + f(2) + f(2.5) + f(3)\right]$$

$$= \frac{1}{2}\left[10.75 + 16 + 22.75 + 31\right]$$

$$= 1/2\left[80.5\right]$$

$$R = 40.25 \text{ units}^2$$

Definite Integrals

$$\int_{-2}^{5}(4x+3)\,dx$$

$$= \frac{\cancel{4}^{\,2}x^2}{\cancel{2}} + 3x + C\Big]_{-2}^{5}$$

$$= (50+15+\cancel{C}) + (-8+6-\cancel{C})$$

$$= 63$$

$$\int_{1}^{6} x\sqrt{x+3}\,dx \qquad u = x+3 \Rightarrow u-3 = x$$

$$du = dx \qquad u = 1+3 = 4 \qquad u = 6+3 = 9$$

change limits

$$\int_{4}^{9}(u-3)\,u^{1/2}\,du$$

$$\int_{4}^{9}\left(u^{3/2} - 3u^{1/2}\right)du$$

$$\frac{2}{5}u^{5/2} - \frac{2\cdot\cancel{3}}{\cancel{3}}u^{3/2}\Big|_{4}^{9}$$

$$= \frac{2}{5}\cdot \underset{\sqrt{9^5}}{9^{5/2}} - 2\cdot \underset{\sqrt[2]{9^3}}{9^{3/2}} + \left(-\frac{2}{5}\cdot 4^{5/2} + 2\cdot 4^{3/2}\right)$$

$$= \frac{2}{5}\cdot 243 - 2\cdot 27 - \frac{2}{5}\cdot 32 + 2\cdot 8$$

$$= \frac{486}{5} - 54 - \frac{64}{5} + 16$$

$$= \frac{422}{5} - 38$$

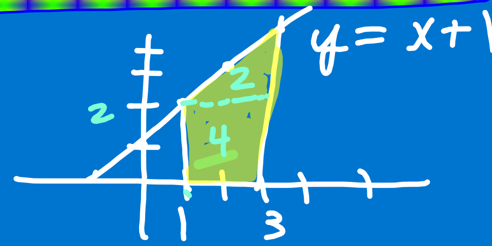
$$= \frac{422}{5} - \frac{190}{5}$$

$$= \frac{232}{5}$$

Fundamental Theorem of Calculus

Part 1

$$\int_1^3 (x+1)\,dx$$
$$= \frac{x^2}{2} + x \Big|_1^3$$
$$= \frac{9}{2} + 3 - \left(\frac{1}{2} + 1\right)$$
$$= 6$$

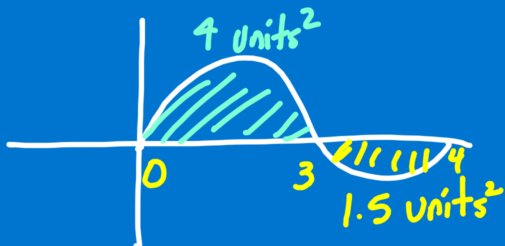


$\Delta = \frac{1}{2} \cdot 2 \cdot 2 = 2$

Area = 6

Integration represents the area between a curve and an axis.

$$\lim_{\Delta x \to 0} \sum_{x=a}^{b} f(x) \cdot \Delta x = \int_a^b f(x)\,dx$$



$$\int_0^3 f(x)\,dx = 4$$
$$\int_0^4 f(x)\,dx = 5.5$$
$$\int_3^0 f(x)\,dx = -\int_0^3 f(x)\,dx = -4$$

Part 2

$$\frac{d}{dx}\int_1^x (4t^2+t)\,dt =$$

$$\frac{4t^3}{3}+\frac{t^2}{2}\Big|_1^x = \frac{d}{dx}\left[\frac{4x^3}{3}+\frac{x^2}{2}-\left(\frac{4}{3}+\frac{1}{2}\right)\right]$$

$$4x^2+x$$

$$\frac{d}{dx}\int_6^x \frac{\sin^8(3t^2-1)}{\ln 8t^4}\,dt = \frac{\sin^8(3x^2-1)}{\ln 8x^4}$$

$$\frac{d}{dx}\int_2^{x^2} \frac{-4t}{\sqrt{t^3+2}}\,dt = \frac{-4x^2}{\sqrt{x^6+2}}\cdot 2x = \frac{-8x^3}{\sqrt{x^6+2}}$$

$$\frac{d}{dx}\int_{x^4}^{3x^7} \frac{2t}{t+1}\,dt$$

$$= \frac{2(3x^7)}{3x^7+1}\cdot 21x^6 - \left(\frac{2x^4}{x^4+1}\cdot 4x^3\right)$$