Business Appl. of Diff. Eq.

The marginal revenue for digital watches is expressed by $\int \frac{dR}{dx} = \int 60{,}000 - \frac{40000}{x^2} x^{-2}$ dollars per thousand. If total sales revenue is $38,000 when 1000 watches are sold, what will revenue be for 4000 watches?

$$R(x) = 60000x + \frac{40000x^{-1}}{-1} + C$$

$$38000 = 60000(1) + \frac{40000}{(1)} + C$$

$$38000 = 100{,}000 + C$$

$$-62{,}000 = C$$

$$R(x) = 60000x + \frac{40000}{x} - 62{,}000$$

$$R(4) = 60{,}000(4) + \frac{40000}{4} - 62000$$

$$= 240{,}000 + 10{,}000 - 62{,}000$$

$$= \$188{,}000$$

(b) $C(x) = 2000x^2 + \frac{40000}{x} + 20{,}000$

How many watches sold to maximize profit?
Max production is 20,000 watches.

$P = R - C$

$P = \left[60000x + \frac{40000}{x} - 62{,}000\right] + \left[-2000x^2 + \frac{40000}{x} - 20000\right]$

$P = -2000x^2 + 60{,}000x - 82{,}000$ Optimization

$P' = -4000x + 60{,}000 = 0$

$\frac{60{,}000}{4000} = \frac{4000x}{4000}$

$15 = x$

Produce 15,000 watches
$P = 368{,}000$

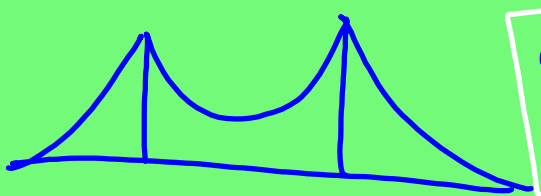
$[0, 20]$

x	
0	-82,000
15	368,000
20	318,000

Hyperbolic Functions

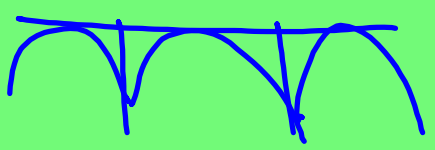
- Combinations of e^x & e^{-x}
- properties of trig functions
- connected through complex numbers

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$



Catenary

$$y = a\cosh\left(\frac{x}{a}\right) + c$$



$$\sinh(\ln 3)$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$= \frac{e^{\ln 3^2} - e^{-\ln 3^{-1}}}{2}$$

$$= \frac{3 - \frac{1}{3}}{2} = \frac{\frac{8}{3}}{\frac{2}{1}} = \boxed{\frac{4}{3}}$$

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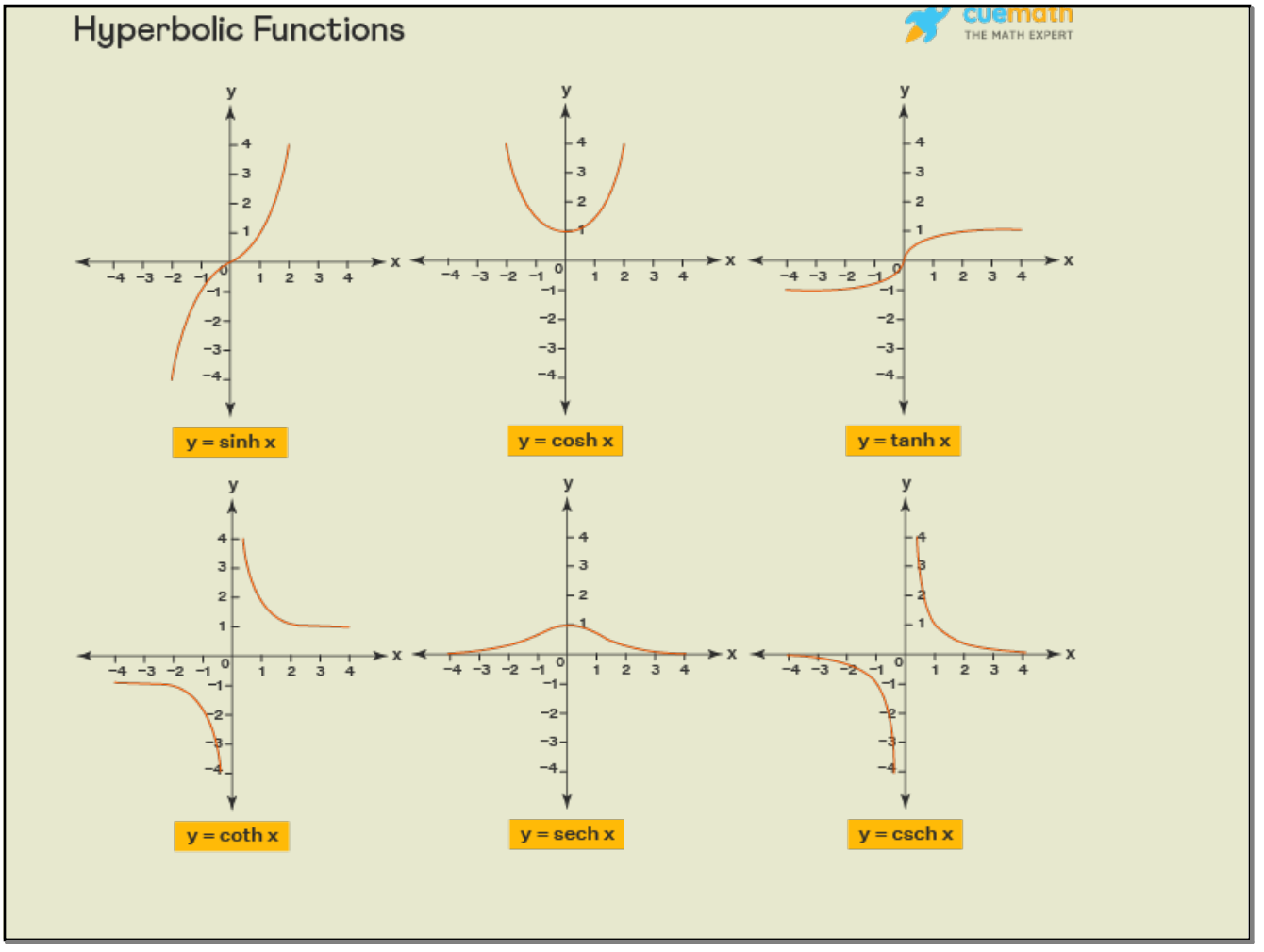

Derivatives

$\frac{d}{dx}\sinh x = \cosh x$ $\quad *\frac{d}{dx}\cosh x = \sinh x$

$\frac{d}{dx}\tanh x = \operatorname{sech}^2 x$ $\quad \frac{d}{dx}\coth x = -\operatorname{csch}^2 x$

$*\frac{d}{dx}\operatorname{sech} x = -\operatorname{sech} x \tanh x$ $\quad \frac{d}{dx}\operatorname{csch} x = -\operatorname{csch} x \coth x$

$\cosh^2 x - \sinh^2 x = 1$

$f(x) = \coth x \cdot e^{\operatorname{csch} x^3}$ Find $f'(x)$

$f'(x) = \coth x \cdot e^{\operatorname{csch} x^3} \cdot -\operatorname{csch} x^3 \coth x^3 \cdot 3x^2$

$+ e^{\operatorname{csch} x^3} \cdot -\operatorname{csch}^2 x$

$$\int \sinh^7 x \cosh x \, dx$$

$u = \sinh x$

$du = \cosh x \, dx$

$$\int u^7 \cdot \cancel{\cosh x} \cdot \frac{du}{\cancel{\cosh x}}$$

$$= \frac{u^8}{8} + C$$

$$= \boxed{\frac{\sinh^8 x}{8} + C}$$

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