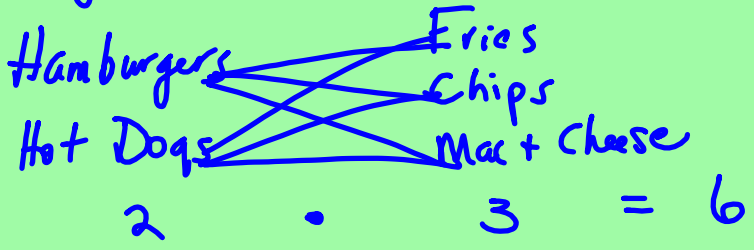
Combinatorics & Probability

of ways to perform an event



$2 \cdot 3 = 6$

Fundamental Counting Principle--If there are p ways to do one event and q ways to do another, then there are p•q ways to do both.

Car manufacturer

- 8 body colors
- 2 fabrics
- 3 option packages

$8 \cdot 2 \cdot 3 = 48$

How many different cars can be made?

Permutations – # of arrangements or patterns that can be formed from a set of objects

Linear Permutations

1) All Objects = $n!$

How many ways are there to ~~arrange the students sitting in the front row?~~ introduce 5 basketball players?

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120 = 5!$

$11 \cdot 10 \cdot 9 \cdot \cdots \cdot 2 \cdot 1 = 11! = 39{,}916{,}800$

factorial

2) Arrange a small group chosen from a larger group $= {}_nP_r$

11 students = 5 front seats

$11 \cdot 10 \cdot 9 \cdot 8 \cdot 7$ $\qquad {}_nP_r = \frac{n!}{(n-r)!}$

${}_nP_r = {}_{11}P_5 = \frac{11!}{(11-5)!} = \frac{11!}{6!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}$

$= 55{,}440$

n = total, r = # to use

How many ways can 4 relay runners be positioned for a race if chosen from 6 possible team members?

${}_6P_4 = \frac{6!}{2!} = 6 \cdot 5 \cdot 4 \cdot 3 = 360$

${}_7P_2 = \frac{7!}{5!} = \frac{7 \cdot 6 \cdot \cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}$

$= \boxed{42}$

3) Alike Objects--indistinguishable, identical

$$\frac{\text{total}!}{\text{alike}!\,\text{alike}!}$$

How many permutations of the letters in the word MISSISSIPPI are possible?

$$\frac{11!}{4!\,4!\,2!} = 34{,}650$$

M I S P

How many ways can 5 identical basketballs, 4 identical volleyballs, and 3 identical soccer balls be arranged in a line?

$$\frac{12!}{5!\,4!\,3!} = 27{,}720$$

4) Specific locations or Repeated objects = Draw blanks

How many ways can six people from this class be arranged in row if there must be a sophomore on each end of the row and two juniors in the middle seats?

$$\underset{So}{6} \cdot 7 \cdot \underset{Jr}{5} \cdot \underset{Jr}{4} \cdot 6 \cdot \underset{Su}{5} = 25{,}200$$

How many different license plates are possible with 3 letters followed by 3 digits letters cannot be repeated but digits can be repeated?

$$\underset{L\ L\ L}{26 \cdot 25 \cdot 24} \cdot \underset{D\ D\ D}{10 \cdot 10 \cdot 10} = 15{,}600{,}000$$

$$26\ 26\ 26 \quad 10\ 10\ 10 = 17{,}576{,}000$$

$$26\ 26\ 26\ 10\ 10\ 10\ 10 = 175{,}760{,}000$$

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COMBINATIONS

— the # of groups that can be formed from a set of objects

↑ no concern for order

$${}_nC_r = \frac{n!}{(n-r)! \cdot r!}$$

n: total, r: # used

$${}_7C_2 = \frac{7!}{5!\,2!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 21$$

Mrs. Meyer wants to form an "I Love Math" Committee to promote mathematics throughout the school. How many different committees of 5 students can be selected from this class?

$${}_{11}C_5 = 462$$

Card Facts

No jokers
52 cards
4 suits – ♡ ♧ ♢ ♤
13 cards in a suit
26 red, 26 black
4 cards of eq. type
12 face cards

Draw 5 cards. How many hands of 5 diamonds are possible?

$${}_{13}C_5 = 1287$$

How many hands with a full house are possible?

3 of a Kind, 2 of a Kind
3 Queens + 2 10's

$${}_{13}C_1 \cdot {}_4C_3 \cdot {}_{12}C_1 \cdot {}_4C_2 = 3744$$

(${}_{13}C_1$: type of card)

$${}_{52}C_5 = 2{,}598{,}960$$

How many hands with 3 black cards and 2 red cards are possible?

$${}_{26}C_3 \cdot {}_{26}C_2 = 845{,}000$$