Applications of Integration

Differential Equations

Complete/general solution have $+C$

particular solution Find C.

$$\int \frac{d^2y}{dx^2} = \int 24x^2 + 18x + 4$$

$$\int \frac{dy}{dx} = \int 8x^3 + 9x^2 + 4x + C_1$$

$$y = 2x^4 + 3x^3 + 2x^2 + C_1x + C_2$$

$$\int \frac{dy^2}{dx^2} = \int (4x+7)^8 \, dx$$

$u = 4x + 7$

$du = 4\,dx$

$$\frac{dy}{dx} = \int u^8 \cdot \frac{du}{4}$$

$4x+7 \downarrow 9$

$$\frac{dy}{dx} = \int \frac{1}{36} u^9 \cdot \frac{du}{4} + \int C_1$$

$$y = \frac{1}{144} \frac{u^{10}}{10} + C_1x + C_2$$

$$y = \frac{1}{1440}(4x+7)^{10} + C_1x + C_2$$

Find a particular solution.

$$\int \frac{d^2y}{dx^2} = \int 3x^2$$

$y = -1$ when $x = 0$

$y' = 9$ when $x = 2$

$$\frac{dy}{dx} = x^3 + C_1$$

$$9 = 2^3 + C_1$$

$$1 = C_1$$

$$\int \frac{dy}{dx} = \int x^3 + 1$$

$$y = \frac{x^4}{4} + x + C_2$$

$$-1 = 0 + 0 + C_2$$

$$\boxed{y = \frac{x^4}{4} + x - 1}$$

Motion

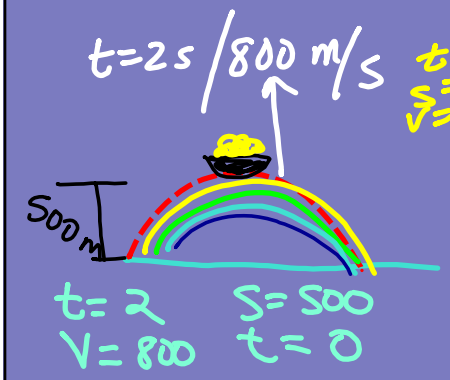
$s(t)$

$v(t) = s'(t)$

$a(t) = v'(t) = s''(t)$

$a = -9.8 \frac{m}{s^2}$

$a = -32 \frac{ft}{s^2}$



$a(t) = -9.8$

What is max height?

$v(t) = -9.8t + C$

$800 = -9.8(2) + C$

$800 = -19.6 + C$

$819.6 = C$

$v(t) = -9.8t + 819.6$

Max height

$0 = -9.8t + 819.6$

$9.8t = 819.6$

$t = 83.63$ sec

$s(t) = -4.9t^2 + 819.6t + C$

$500 = 0 + 0 + C$

$s(t) = -4.9t^2 + 819.6t + 500$

$s = -4.9(83.63)^2 + 819.6(83.63) + 500$

$s = 34,772.66$ m

What is velocity at the moment it strikes the ground?

t=

s = 0

V=

A bicyclist applies brakes + decelerate at 2 ft/s². How far will he travel before stopping if his speed has reduced to 6 ft/s after 2 sec?

$a(t) = -2$

$V(t) = -2t + C$

$6 = -2(2) = C$

$10 = C$

$\boxed{V(t) = -2t + 10}$

$S(t) = -t^2 + 10t + C$

$0 = 0 + 0 + C$

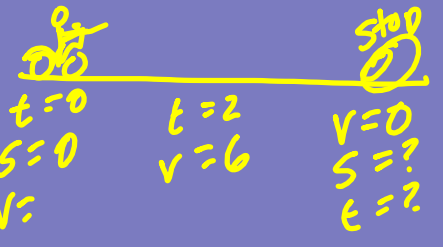
$\boxed{S(t) = -t^2 + 10t}$

$t = 0$, $a = -2$

$t = 2$, $V = 6$

$V = \frac{ft}{sec}$

$a = \frac{V}{t} = \frac{\frac{ft}{sec}}{\frac{sec}{1}}$



$0 = -2t + 10$

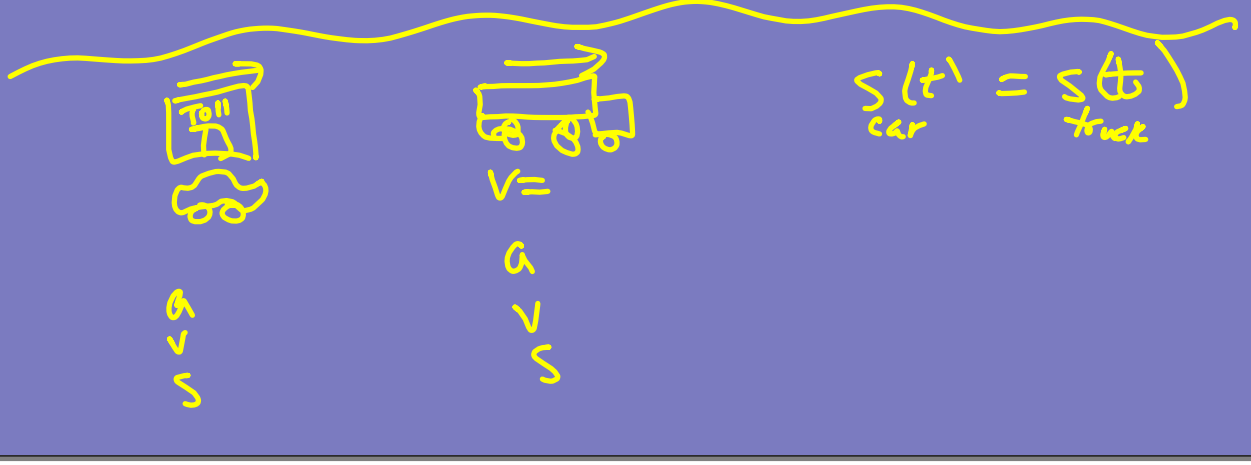
$2t = 10$

$t = 5$ sec

$S(5) = -(5)^2 + 10(5)$

$= -25 + 50$

$= \boxed{25 \text{ ft}}$



Business Applications

Marginal Revenue = change in rev per item sold

$$\frac{dR}{dx} = 60{,}000 - \frac{40{,}000x^{-2}}{x^2}$$ dollars per thousand.

Total sales revenue is $38,000 when 1000 watches are sold. What is revenue for 4000 watches?

(b) Cost of watches

$$C(x) = 2000x^2 + \frac{40000}{x} + 20,000$$

How many watches to optimize profit.

Max production level is 20,000 watches.