March 7, 2025

# Inverse Trig Functions

$y = x^3 + 4$

$x = y^3 + 4$

$\sqrt[3]{x-4} = \sqrt[3]{y^3}$

$\sqrt[3]{x-4} = f^{-1}$

1) Switch x + y
2) Solve for y.

$y = \sin\theta$

$\theta = \sin^{-1} y$

$\theta = \text{Arcsin}\, y$

$\frac{1}{2} = \sin\frac{\pi}{6}$

$\frac{\pi}{6} = \text{Arcsin}\,\frac{1}{2}$

Inverse Trig functions represent angles!

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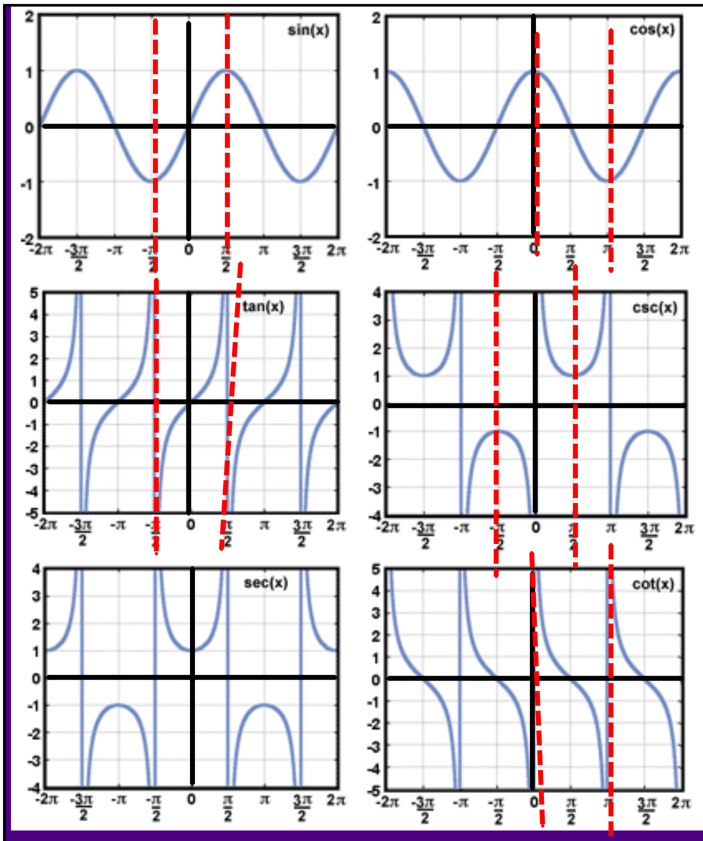
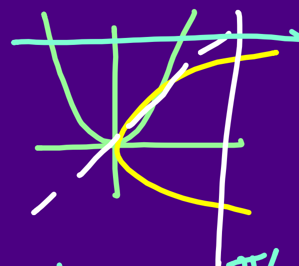


$\pi/2$

$-$ { $\underline{C}os^{-1}x$, $\underline{S}ec^{-1}x$, $\underline{C}ot^{-1}x$ }

$\pi$

All +

0

{ $Csc^{-1}x$, $Sin^{-1}x$, $Tan^{-1}x$ } $-$

$-\pi/2$

Capitalized = Use limited quadrants

| | |
|---|---|
| Cody Sells Cocaine | At |
| | Crazy Sexy Time |

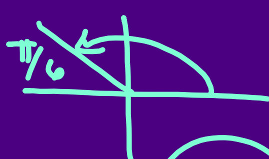
Answers are angles – always in radians!

Cos⁻¹ x, Sec⁻¹ x, Cot⁻¹ x: –   |   All +

Csc⁻¹ x, Sin⁻¹ x, Tan⁻¹ x: –

$$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$


$$\cot^{-1}\left(-\sqrt{3}\right)$$

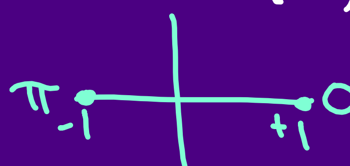



$$\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$$

$$\frac{5\pi}{6}$$

$$\text{Arcsin}\left(-\frac{\sqrt{3}}{2}\right) = \boxed{\frac{-\pi}{3}}$$

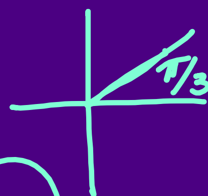

$$\text{Arcsec}(-1) = \boxed{\pi}$$



$\cos(\underbrace{\operatorname{Tan}^{-1}\sqrt{3}}_{\theta})$

$\cos\ \theta$

$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$




$\sec\left(\operatorname{Arccsc}\frac{x}{4}\right) = \frac{r}{y}$

$\sec(\theta) = \frac{r}{x}$

$= \frac{x}{\sqrt{x^2-16}}$

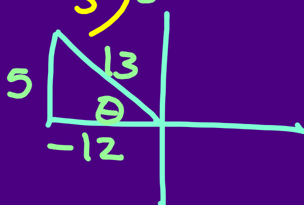

$a = \sqrt{x^2-16}$

$a^2 + 16 = x^2$

$\sqrt{a^2} = \sqrt{x^2-16}$

$\sin(\operatorname{Arccot} -\frac{12}{5})\ \frac{x}{y}$

$\sin(\theta)$




$= \frac{y}{r}$

$= \boxed{\frac{5}{13}}$

$25 + 144 = r^2$

$\sqrt{169} = \sqrt{r^2}$

$13 = r$

$\tan(\underbrace{\operatorname{Cos}^{-1}\left(-\frac{7}{9}\right)}_{\tan\ \theta})\ \frac{x}{r}$


$y^2 + 49 = 81$

$\sqrt{y^2} = \sqrt{32}$

$y = \pm 4\sqrt{2}$

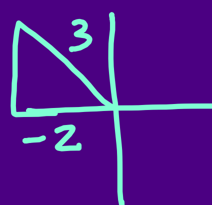
$\tan\theta = \frac{y}{x} = \frac{4\sqrt{2}}{-7}$

$\sin\left(2\underbrace{\text{Arccos}\left(-\frac{2}{3}\right)}\right)$ $\frac{x}{r}$

$\sin(2\theta)$

$= 2\sin\theta\cos\theta$

$\frac{y}{r}$ $\frac{x}{r}$




$y^2+4=9$

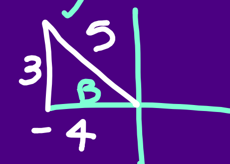
$\sqrt{y^2}=\sqrt{5}$

$= 2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{-2}{3}\right)$

$= \boxed{\frac{-4\sqrt{5}}{9}}$

$\cos\left(\underbrace{\text{Arctan}\left(\frac{1}{3}\right)}\frac{y}{x} - \underbrace{\text{Arcsec}\left(\frac{-5}{4}\right)}\frac{r}{x}\right)$

$\cos(A - B)$


$9+1=r^2$

$\sqrt{10}=\sqrt{r^2}$

$y^2+16=25$

$\sqrt{y^2}=\sqrt{9}$

$\cos A\cos B + \sin A\sin B$

$\left(\frac{3}{\sqrt{10}}\right)\left(\frac{-4}{5}\right)+\left(\frac{1}{\sqrt{10}}\right)\left(\frac{3}{5}\right)$

$\frac{-12}{5\sqrt{10}}+\frac{3}{5\sqrt{10}}$

$= \frac{-9}{5\sqrt{10}}$

# INVERSE TRIG EQUATIONS ← Has = sign

Solve for x.

$y = 3\sin\left(\frac{2}{3}x\right) - 4$

$\frac{y+4}{3} = \frac{3\sin\left(\frac{2}{3}x\right)}{3}$

$\frac{y+4}{3} = \sin\left(2/3\,x\right)$

$\cancel{\frac{3}{2}}\cancel{\frac{2}{3}}x = \frac{3}{2}\sin^{-1}\left(\frac{y+4}{3}\right)$

$x = \frac{3}{2}\sin^{-1}\left(\frac{y+4}{3}\right)$

1) Isolate Trig func.
2) Switch variables using an inverse.
3) Check if needed

$3\pi + 4\,\mathrm{Tan}^{-1} y = 2\pi$
$-3\pi \qquad\qquad -3\pi$

$\frac{4\,\mathrm{Tan}^{-1} y}{4} = \frac{-\pi}{4}$

$\mathrm{Tan}^{-1} y = -\pi/4$ ← Check quadrant

$\tan\left(-\pi/4\right) = y$


$-1 = y$