March 28, 2025

Polar Coordinates & Complex Numbers

Complex Numbers

$(3+4i)+(2-7i) = 5-3i$

$(7+2i)(1-5i)$ FOIL

$= 7-35i+2i+10i^2$

$= \boxed{17-33i}$

$\frac{4+2i}{3-4i}\cdot\frac{(3+4i)}{(3+4i)}$

$= \frac{12+16i+6i+8i^2}{9+16i^2} =$

$= \frac{4+22i}{25} = \frac{4}{25}+\frac{22i}{25}$

$i = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i = \sqrt{-1}$

$i^{47} = -i$ $\frac{47}{4} = 11.75$ $\frac{3}{4}$

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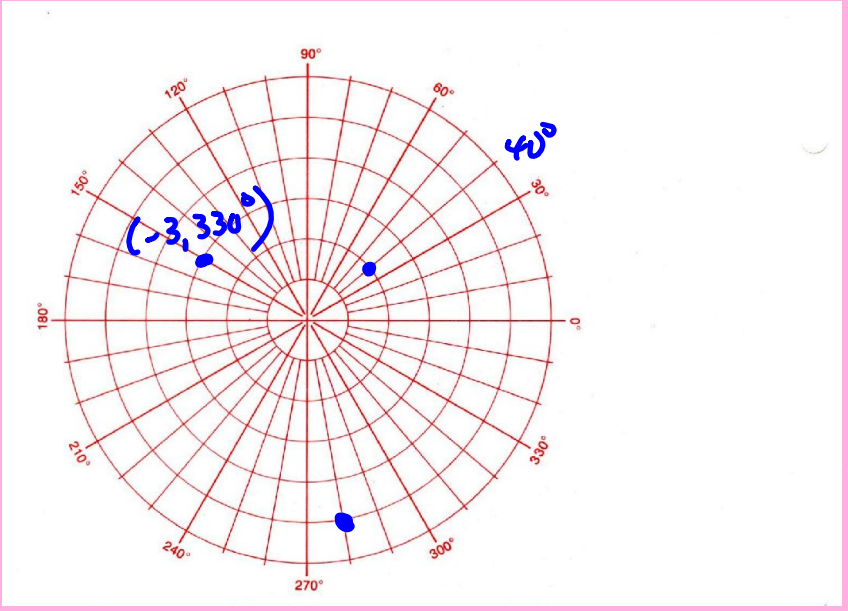

Polar Coord.

(r, θ)

$(2, 40^\circ)$

$(5, 280^\circ)$

$(-3, 330^\circ)$

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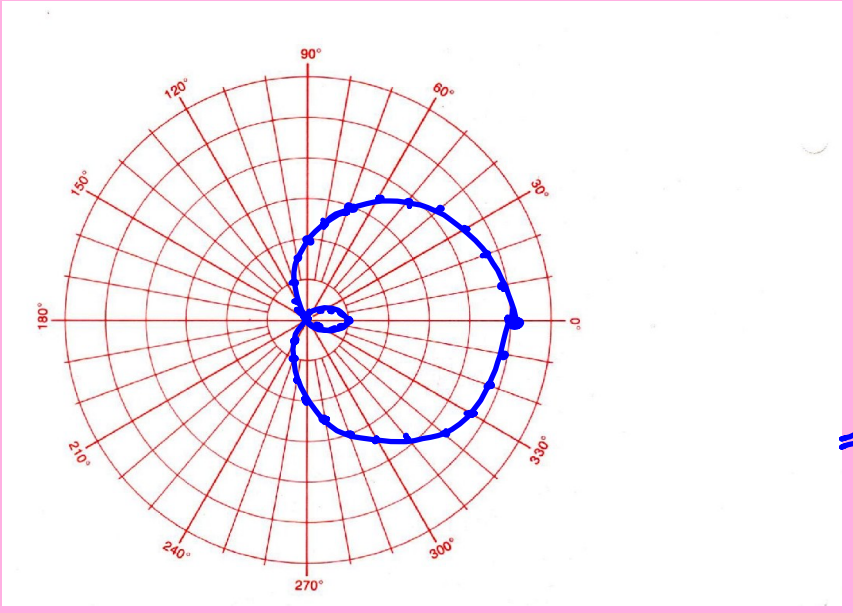


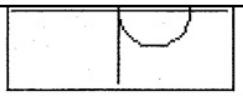
$r = 2 + 3\cos\theta$

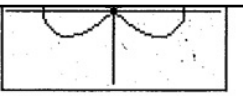
θ	r
$0°$	5
$10°$	
$20°$	

limaçon

↓

cardioid

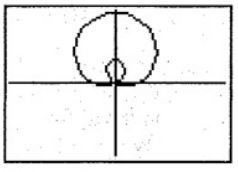
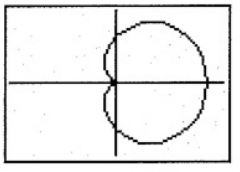
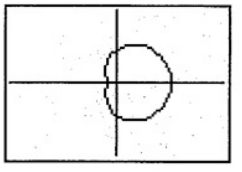
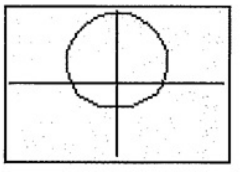


$r = a \cos \theta$ $r = a \sin \theta$ $r^2 = a^2 \sin 2\theta$ $r^2 = a^2 \cos 2\theta$

Limaçons

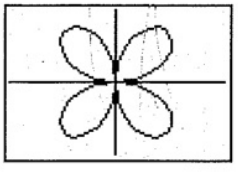
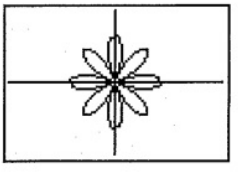
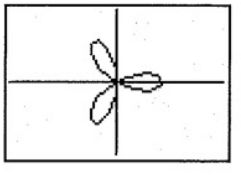
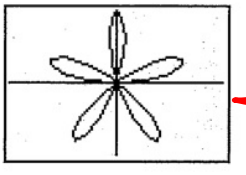
$r = a \pm b \sin \theta$ or $r = a \pm b \cos \theta$

$\frac{a}{b} < 1$ $\frac{a}{b} = 1$ $1 < \frac{a}{b} < 2$ $\frac{a}{b} \geq 2$

Rose Curves

$2n$ petals if n is even, $n \geq 2$ n petals if n is odd

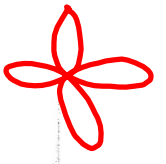





$n = 2$
$r = a \sin n\theta$

$n = 4$
$r = a \cos n\theta$

$n = 3$
$r = a \cos n\theta$

$n = 5$
$r = a \sin n\theta$

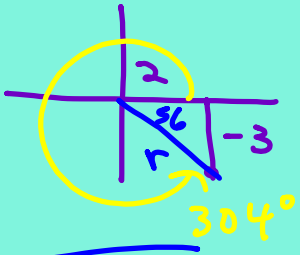
Converting between Equation Forms Sometimes an equation given in polar form is easier to graph in rectangular (Cartesian) form. To convert a polar equation to a rectangular equation, we use the following relationships, which were introduced in Section 8.2. See triangle POQ in Figure 36.

Converting Coordinates

Rectangular (x, y)

Polar (Trigonometric) (r, θ)

$(2, -3)$ Convert to polar.



$(\sqrt{13}, 304°)$

$2^2 + (-3)^2 = r^2$

$4 + 9 = \sqrt{13} = \sqrt{r^2}$

$\tan\theta = -3/2$

$\theta = \tan^{-1}(3/2)$

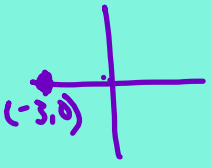
$= 56°$

Rect → Polar

$x^2 + y^2 = r^2$

$\tan\theta = \frac{y}{x}$

$(-3, 0)$



$r = 3$

$\theta = 180°$

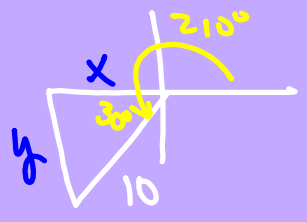
$(3, 180°)$

Polar → Rect

$(r, \theta) \longrightarrow (x, y)$

$x = r\cos\theta$
$y = r\sin\theta$

$(10, 210°)$



$\cos 210° = \frac{x}{10}$ $\qquad$ $\sin 210° = \frac{y}{10}$

$10\cos 210° = x$ $\qquad$ $10\sin 210° = y$

$10\left(\frac{\sqrt{3}}{2}\right) = x$ $\qquad$ $10\left(-\frac{1}{2}\right) = y$

$-5\sqrt{3} = x$ $\qquad$ $-5 = y$

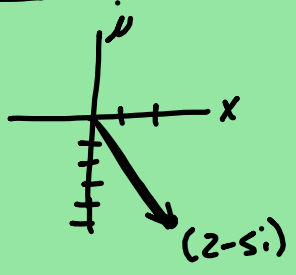
$(-5\sqrt{3}, -5)$

Rectangular

$x + yi$

$2 - 5i$

$(3+8i)^4$



Polar Form (Trigonometric)

$r\cos\theta + r\sin\theta i$

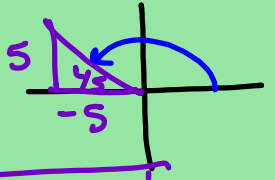
$* r(\cos\theta + i\sin\theta)$

$\sqrt[3]{x^3} = \sqrt[3]{8}$

$x = 2$

Convert $-5 + 5i$ to polar form.

(x y)



$r^2 = 25 + 25$

$\sqrt{r^2} = \sqrt{50}$

$r = 5\sqrt{2}$

$\tan\theta = \frac{5}{-5} = -1$

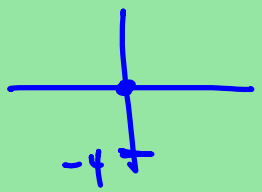
$\theta = 135^\circ$

$r(\cos\theta + i\sin\theta)$

$5\sqrt{2}(\cos 135^\circ + i\sin 135^\circ)$

$0 - 4i$

Write as a complex # in polar form



$r = 4$

$\theta = 270^\circ$

$4(\cos 270^\circ + i\sin 270^\circ)$

$4\,\mathrm{cis}\,270^\circ$

Polar Form to Rect. Form

$2(\cos 359^\circ + i \sin 359^\circ)$

$x = 2\cos 359^\circ \quad y = 2\sin 359^\circ$

$= 1.999659 \quad y = -0.0349$

$1.999659 - 0.0349i$

$4(\cos 240^\circ + i \sin 240^\circ)$

$4\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$= -2 - 2i\sqrt{3}$