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Solving Trig Equations 2

$(2\sin x)^2 = (1-2\cos x)^2 \quad [0°, 360°)$

$4\sin^2 x = (1-2\cos x)(1-2\cos x) \quad \sin^2 x + \cos^2 x = 1$

$4\sin^2 x = 1 - 2\cos x - 2\cos x + 4\cos^2 x$

$4(1-\cos^2 x) = 4\cos^2 x - 4\cos x + 1$

$4 - 4\cos^2 x = 4\cos^2 x - 4\cos x + 1$

$0 = 8\cos^2 x - 4\cos x - 3$

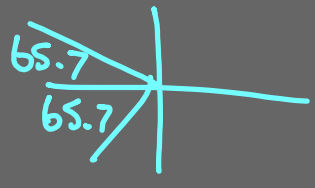
$$\cos x = \frac{4 \pm \sqrt{16 - 4(8)(-3)}}{2(8)}$$

$\cos x = 0.911$

$x = 24.3°, 335.7°$

$\cos x = -0.411$

$x = 65.7$



$114.3°, 245.7°$

Must Check

$2\sin x = 1 - 2\cos x$

Calculator:

$2\sin x \,|\, x = 24.3$

$1 - 2\cos x \,|\, x = 24.3$

Check

1) Trig function in denom

2) Square both sides

Use identities when:

1) Different angles (x, $2x$, $\frac{x}{2}$)
2) Different trig functions

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$\sin x - \sin 2x = 0 \quad [0, 2\pi)$

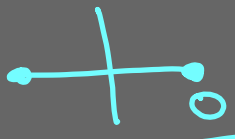
↑ Double angle!

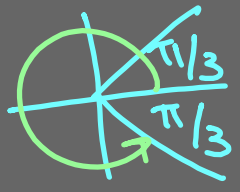
$\sin x - 2\sin x\cos x = 0$

$\sin x(1 - 2\cos x) = 0$

$\sin x = 0 \qquad 1 - 2\cos x = 0$

$\frac{1}{2} = \frac{2\cos x}{2}$





$x = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$

$\sin x - \cos 2x = 0$

$\sin x - [1 - 2\sin^2 x] = 0$

↑ Distribute

$\sin x = \cos \frac{x}{2}$ $\quad [0°, 360°)$

$(\sin x)^2 = \left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2$

$\sin^2 x = \frac{1+\cos x}{2}$

$2\left[1-\cos^2 x = \frac{1+\cos x}{2}\right]$

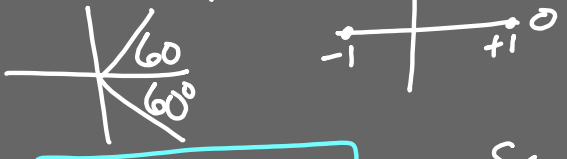
$2 - 2\cos^2 x = 1 + \cos x$

$0 = 2\cos^2 x + \cos x - 1$

$0 = (2\cos x - 1)(\cos x + 1)$

$2\cos x - 1 = 0$ $\quad \cos x + 1 = 0$

$\cos x = \frac{1}{2}$ $\quad \cos x = -1$



$60°, 300°, 180°$

Squared both sides!
Check!

$\sin x = \cos \frac{x}{2}$

$60°$ $\quad \sin 60° = \cos 30°$

$\sqrt{3}/2 = \sqrt{3}/2$

$\sin 300° = \cos 150°$

$-\sqrt{3}/2 = -\sqrt{3}/2$

$\sin 180° = \cos 90°$

$0 = 0$

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$2\cos^2 3\theta + 3\cos 3\theta + 1 = 0$

$\theta = [0°, 360°)$

$3\theta = [0°, 1080°)$

$(2\cos 3\theta + 1)(\cos 3\theta + 1) = 0$ If:

1) all same trig func
2) all same multiple angle.

$\cos 3\theta = \frac{-1}{2}$ $\cos 3\theta = -1$

$3\theta = 120°, 240°, 180°$

$+360$ $480°, 600°, 540°$

$+360$ $840°, 960°, 900°$

$\rightarrow$

$\theta = 40°, 80°, 60°$

$= 160°, 200°, 180°$

$= 280°, 320°, 300°$

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$2\tan\left(\frac{x}{2}\right) + \sqrt{3} = -\tan\frac{x}{2}$

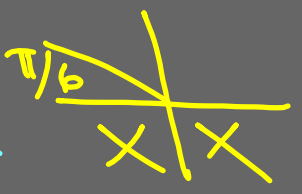
$+\tan(x/2)$

$\frac{3\tan\left(\frac{x}{2}\right)}{3} = \frac{-\sqrt{3}}{3}$

$x = [0, 2\pi)$

$\frac{x}{2} = [0, \pi)$

$\tan\left(\frac{x}{2}\right) = \frac{-\sqrt{3}}{3}$



$2 \cdot \frac{x}{2} = \frac{5\pi}{6} \cdot 2$

$x = \frac{5\pi}{3}$