

# SEQUENCES + SERIES

Sequences - a list of #s that follow a pattern.

1, 1, 2, 3

Arithmetic

1, 2, 3, 4, 5, ...

10, 20, 30, 40, ...

Geometric

2, 4, 8, 16, 32, ...

Series - the sum of the #s in a sequence

Leonardo de Fibonacci

0, 3, 8, 15, 24, ...  
 $1^2 - 1$ ,  $2^2 - 1$ ,  $3^2 - 1$ ,  $4^2 - 1$ ,  $5^2 - 1$

## FIBONACCI SEQUENCE

1, 1, 2, 3, 5, 8, 13, 21, 34, ...  
 55, 89

$a_1$ ,  $a_2$ ,  $a_3$

first term

$a_n$

last term or unknown term

$n = \#$  of terms

Find the first 4 terms.

$$a_n = 4n + 2$$

$$a_1 = 4(1) + 2 = 6$$

$$a_2 = 4(2) + 2 = 10$$

$$a_3 = 4(3) + 2 = 14$$

$$a_4 = 4(4) + 2 = 18$$

$$a_n = \frac{n+2}{2n}$$

$$a_1 = \frac{1+2}{(2)(1)} = \frac{3}{2}$$

$$a_2 = \frac{2+2}{2(2)} = \frac{4}{4} = 1$$

$$a_3 = \frac{3+2}{2(3)} = \frac{5}{6}$$

$$a_4 = \frac{4+2}{2(4)} = \frac{6}{8} = \frac{3}{4}$$

# SUMMATION NOTATION

$$\sum_{n=1}^4 (2n-3) = 2(1)-3 \quad 2(2)-3 \quad 2(3)-3 \quad 2(4)-3$$

$$= -1 + 1 + 3 + 5$$

$$= \boxed{8}$$

$$\sum_{j=22}^{50} (4j+7)$$

$a_1$        $a_n$

$$S_n = \frac{n}{2} (a_1 + a_n)$$

$$S_n = \frac{29}{2} (95 + 207)$$

$$= 4379$$

$$\sum_{i=1}^n p_i + q$$

Arithmetic

$$n = 50 - 22 + 1$$

$$n = 29$$

$$a_1 = 4(22) + 7$$

$$= 95$$

$$a_n = 4(50) + 7$$

$$= 207$$

# ARITHMETIC SEQUENCES = adds the same value to each term

1, 2, 3, 4, 5, ...

2, 4, 6, 8, 10, ...

2.4, 3.6, 4.8, 6.0  
 $d = 1.2$

100, 93, 86, 79, ...  
 $d = 93 - 100 = -7$

Common  
difference  
 $= d$   
 $d = a_2 - a_1$

3, 11, 19, 27, ...  $d = 8$  Find the 200<sup>th</sup> term.

$$3 + 199(8) = 1595$$

$$a_n = a_1 + d(n-1)$$

$\frac{17}{12}, \frac{5}{6}, \frac{1}{4}, \dots$  Find 8<sup>th</sup> term.

$\frac{17}{12}, \frac{10}{12}, \frac{3}{12}$       $d = a_2 - a_1$   
 $d = -\frac{7}{12}$

$$a_8 = \frac{17}{12} + \left(-\frac{7}{12}\right)(8-1)$$

$$= \frac{17}{12} + -\frac{49}{12}$$

$$= -\frac{32}{12} = \boxed{-\frac{8}{3}}$$

# ARITHMETIC SERIES (Sum of all terms)

$$S_4 = 5 + 8 + 11 + 14 \quad \boxed{= 38} \quad S_n = \text{sum of terms}$$

$$+ \quad \underline{S_4 = 14 + 11 + 8 + 5}$$

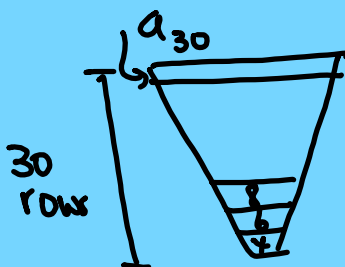
$$2S_4 = 19 + 19 + 19 + 19$$

$$2S_4 = 4(19)$$

$$\frac{2S_4}{2} = \frac{76}{2}$$

$$S_4 = 38$$

$$\longrightarrow S_n = \frac{n}{2}(a_1 + a_n)$$



How many seats in top row?

$$a_n = a_1 + d(n-1)$$

$$\begin{aligned} a_{30} &= 4 + 2(30-1) \\ &= 4 + 58 \\ &= 62 \end{aligned}$$

How many seats in the whole section?

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_{30} = \frac{30}{2}(4 + 62)$$

$$= 990 \text{ seats}$$

Find  $S_n$ .

$$52 + 64 + 76 + \dots + 1816.$$

$$d = 12$$

$$S_n = \frac{n}{2} (a_1 + a_n)$$

$$S_n = \frac{n}{2} (52 + 1816)$$

$$S_n = \frac{148}{2} (1868)$$

$$S_n = 74 (1868)$$

$$S_n = 138,232$$

$$a_n = a_1 + d(n-1)$$

$$1816 = 52 + 12(n-1)$$

$$\begin{array}{r} 1816 \\ -52 \\ \hline 1764 \end{array} = \begin{array}{r} 12(n-1) \\ -52 \\ \hline 1764 \end{array}$$

$$147 = n-1$$

$$148 = n$$

