GEOMETRIC SEQUENCES – multiply by the same value

$3, 12, 48, 192, \ldots$

r = common ratio $= \frac{a_2}{a_1}$

$81, 54, 36, 24, \ldots$

$r = \frac{54}{81} = \frac{6}{9} = \frac{2}{3}$

$5, -15, 45, -135, \ldots$

$r = \frac{-15}{5} = -3$

$2, 6, 18, 54, \ldots$ (a_2, a_3, a_4)

$\times 3$ (2×3)

2×9 $\quad 2\cdot 27$

2×3^1 $\quad 2\times 3^2$ $\quad 2\cdot 3^3$

$$a_n = a_1 \cdot r^{n-1}$$

Find 8th term.

$a_n = 2\cdot 3^{8-1}$ $\quad 2187$

$= 4374$

$150, -60, 24, \ldots$

Find a_{11}.

$r = \frac{-60}{150} = -\frac{2}{5}$

$a_{11} = 150 \cdot \left(-\frac{2}{5}\right)^{11-1}$

$= \frac{6144}{390{,}625}$

Population of Zeno is decreasing by 8% per year. The population is currently 2100. What will the pop. be in 7 years?

$2100, \ldots, a_8$

a_1

$a_n = a_1 \cdot r^{n-1}$

$a_8 = 2100(0.92)^{8-1}$

≈ 1171 people

100%
− 8%
92%

$r = 0.92$

Pop. inc. by 5%

100%
+5%
105%

$r = 1.05$

Geometric Series

Sum

$1S_n = 2 + 10 + 50 + 250 + \ldots$

$-5S_n = \quad 10 + 50 + 250 + 1250 + \ldots$

$-4S_n = 2 - 1250$

$\frac{-4S_n}{-4} = \frac{-1248}{-4}$

$S_n = 312$

$S_n = 312$

$r = 5$

$$S_n = \frac{a_1 - a_n \cdot r}{1 - r}$$

Know a_n (last term)

$$S_n = \frac{a_1 - a_1 \cdot r^{n-1} \cdot r^1}{1 - r}$$

$$S_n = \frac{a_1 - a_1 \cdot r^n}{1 - r}$$

Know n (# of terms)

Find S_n.

$6 + 24 + 96 + \cdots + 6{,}291{,}456$ (a_n)

$$S_n = \frac{a_1 - a_n \cdot r}{1 - r} \qquad r = \frac{24}{6} = 4$$

$$= \frac{6 - (6291456)(4)}{1 - 4} = 8{,}388{,}606$$

$500 - 300 + 180 - \cdots$ to 10 terms

$$S_n = \frac{a_1 - a_1 \cdot r^n}{1 - r} = \frac{500 - 500\left(-\frac{3}{5}\right)^{10}}{1 + \frac{3}{5}}$$

$r = \frac{-300}{500}$

$$= \frac{4853288}{15625}$$

$$\sum_{K=2}^{9} 7 \cdot 3^{K-2} = 7 \cdot 3^{2-2} + 7 \cdot 3^{3-2} + 7 \cdot 3^{4-2}$$

$$= 7 + 21 + 63 + \cdots$$

$a_1 = 7 \cdot 3^{2-2} = 7$

$n = 9 - 2 + 1$

$n = 8$

$$S_n = \frac{a_1 - a_1 \cdot r^n}{1 - r}$$

$$= \frac{7 - 7 \cdot 3^{8}}{1 - 3}$$

$$= \boxed{22{,}960}$$

$\sum p \cdot q^i$, $r = q$

calculator $(-3)^8$

Infinite Geometric Series

$2 + 10 + 50 + 250 + \cdots = \infty$ diverges $|r| \geq 1$

$r = 5$

$250 + 50 + 10 + 2 + \frac{2}{5} + \frac{2}{25} =$ Converges (does not go to ∞) $0 \leq |r| \leq 1$

$r = \frac{1}{5}$

$$S_n = \frac{a_1 - a_1 \cdot r^n}{1 - r} = \frac{250 - 250\left(\frac{1}{5}\right)^{\infty}}{1 - \frac{1}{5}}$$ (∞ → 0)

$$S = \frac{a_1}{1 - r}$$

$$= \frac{250}{4/5} = \frac{1250}{4}$$