INTEGRATION BY PARTS

Integration by Parts

$\int x^2 e^x dx$

when multiplying two unrelated func.

$$\int (f \cdot g)' = \int \underline{f \cdot g'} + \int g \cdot f'$$

$$f \cdot g - \int g \cdot f' = \int f \cdot g'$$

$$u \cdot v - \int v \cdot du = \int u \cdot dv$$

$$\int u\, dv = uv - \int v\, du$$

$$\int x \sec^2 x \, dx \qquad \begin{array}{ll} u = x & dv = \sec^2 x \, dx \\ du = dx & v = \tan x \end{array} \qquad \int u \, dv = uv - \int v \, du$$

$$= x \tan x - \int \tan x \, dx$$

$$= x \tan x - \int \frac{\sin x}{\cos x} \, dx \qquad \begin{array}{l} u = \cos x \\ du = -\sin x \, dx \end{array}$$

$$= x \tan x - \int \frac{\cancel{\sin x}}{u} \frac{du}{-\cancel{\sin x}}$$

$$= x \tan x + \int \frac{1}{u} \, du$$

$$= x \tan x + \ln|u| + C$$

$$= \boxed{x \tan x + \ln|\cos x| + C}$$

$\int x^2 e^{2x} dx$

$u = x^2 \quad dv = e^{2x}$

$du = 2x\,dx \quad v = \frac{1}{2}e^{2x}$

$u = 2x$

$du = 2dx$

$\int e^u \cdot \frac{du}{2}$

$\frac{1}{2}e^{2x}$

$= \frac{1}{2}x^2e^{2x} - \int xe^{2x}dx$

$u = x \quad dv = e^{2x}dx$

$du = dx \quad v = \frac{1}{2}e^{2x}$

$= \frac{1}{2}x^2e^{2x} + \left[-\frac{1}{2}xe^{2x} + \int \frac{1}{2}e^{2x}dx\right]$

$= \boxed{\frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C}$

$\int \cos 3x$

$\frac{1}{3}\sin 3x$

$\int e^x \cos x \, dx$

$u = e^x \quad dv = \cos x \, dx$

$du = e^x dx \quad v = \sin x$

$e^x \sin x - \int e^x \sin x \, dx$

$u = e^x \quad dv = \sin x \, dx$

$du = e^x \quad v = -\cos x$

$\int e^x \cos x \, dx = e^x \sin x + [+ e^x \cos x \mp \int + e^x \cos x$

$2 \int e^x \cos x \, dx = e^x \sin x + e^x \cos x$

$\int e^x \cos x \, dx = \dfrac{e^x \sin x + e^x \cos x}{2} + C$

$\int \ln x \, dx$

$u = \ln x \quad dv = dx$

$du = \frac{1}{x} dx \quad v = x$

$x \ln x - \int 1 \, dx$

$\boxed{x \ln x - x + C}$