Operations in Polar Form

$(2+3i)(4\cdot 7i)$

$2(\cos 30^\circ + i\sin 30^\circ)\cdot 5(\cos 70^\circ + i\sin 70^\circ)$

$10(\underline{\cos 30^\circ \cos 70^\circ} + \underline{i\cos 30^\circ \sin 70^\circ} + \underline{i\sin 30^\circ \cos 70^\circ} \mp \cancel{i^2}\,\underline{\sin 30^\circ \sin 70^\circ})$

$10(\cos(30^\circ + 70^\circ) + i\sin(30^\circ + 70^\circ))$

$10(\cos 100^\circ + i\sin 100^\circ)$

$r_1(\cos\theta_1 + i\sin\theta_1)\cdot r_2(\cos\theta_2 + i\sin\theta_2) =$

$r_1 r_2[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)]$

$37(\cos 211^\circ + i\sin 211^\circ)\cdot 4(\cos 348^\circ + i\sin 348^\circ)$

$= \overset{37\cdot 4}{148}[\cos \overset{211+348}{559^\circ} + i\sin 559^\circ]$

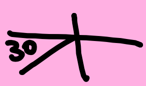
$$\frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} = \frac{r_1}{r_2}\left[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right]$$

Divide & change to rectangular form.

$$\frac{15(\cos 340^\circ + i\sin 340^\circ)}{3(\cos 550^\circ + i\sin 550^\circ)}$$

$$= 5(\cos(340^\circ - 550^\circ) + i\sin(340^\circ - 550^\circ)]$$

$$= 5[\cos(-210^\circ) + i\sin(-210^\circ)]$$



$$= 5\left[-\frac{\sqrt{3}}{2} + i\frac{1}{2}\right]$$

$$= -\frac{5\sqrt{3}}{2} + \frac{5}{2}i$$

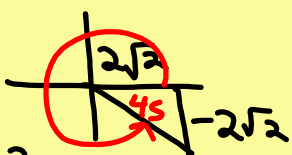
DeMoivre's Theorem

$$[r(\cos\theta + i\sin\theta)]^3 = r^3(\cos(3\theta) + i\sin(3\theta))$$

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos(n\cdot\theta) + i\sin(n\theta))$$

$$(2\sqrt{2} - 2i\sqrt{2})^6$$

1) Change to polar form
2) Use DeMoivre's theorem to raise it to a power
3) Convert back to rect. form



$$\underset{4\cdot 2}{(2\sqrt{2})^2} + (2\sqrt{2})^2 = r^2$$

$$8 + 8 = r^2$$

$$16 = r^2$$

$$4 = r$$

$$\tan\theta = \frac{-\cancel{2\sqrt{2}}}{\cancel{2\sqrt{2}}} = -1$$

$$\theta = 315^\circ$$

$$[4(\cos 315^\circ + i\sin 315^\circ)]^6$$

4^6 $\quad 315^\circ \cdot 6$

$$\frac{1890}{360} = 5.25$$

$$360 \cdot 5 = 1800^\circ$$

$$= 4096(\cos \cancel{1890}\,90^\circ + i\sin \cancel{1890}\,90^\circ)$$

$$= 4096(0 + i(1))$$

$$= \boxed{4096i}$$

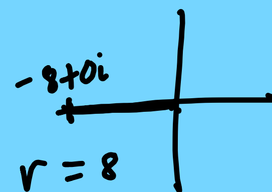
Solve. $x^3+8=0$

$\sqrt[3]{x^3}=\sqrt[3]{-8}$

$x=-2$

$(x^3)^{1/3}=(-8)^{1/3}$

$x=(-8+0i)^{1/3}$



$r=8$

$\theta=180°$

$$\left[8(\cos 180° + i\sin 180°)\right]^{1/3} = 8^{1/3}\left(\cos\left(\tfrac{1}{3}\cdot 180°\right) + i\sin\left(\tfrac{1}{3}\cdot 180°\right)\right)$$

$$= 2(\cos 60° + i\sin 60°)$$

$$= 2\left(\tfrac{1}{2} + i\tfrac{\sqrt{3}}{2}\right)$$

$$= \boxed{1 + i\sqrt{3}}$$

$+360°$

$$\left[8(\cos 540° + i\sin 540°)\right]^{1/3} = 2(\cos 180° + i\sin 180°)$$

$$= 2(-1 + i0)$$

$$= \boxed{-2}$$

$+360°$

$$\left[8(\cos 900° + i\sin 900°)\right]^{1/3} = 2(\cos 300° + i\sin 300°)$$

$$= 2\left(\tfrac{1}{2} + i\tfrac{-\sqrt{3}}{2}\right)$$

$$= \boxed{1 - i\sqrt{3}}$$

$360°\cdot\tfrac{1}{3} = 120°$

1) Isolate the variable.

2) Eliminate the power on the variable by using the 1/n power.

3) Change to polar form.

4) Apply DeMoivre's Theorem.

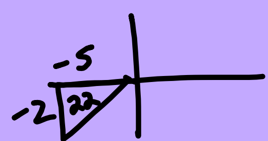
5) Get additional answers by taking $(1/n) \cdot 360^\circ$ and add to first answer.

$$x^4 - (-5-2i) = 0$$

$$(x^4)^{1/4} = (-5-2i)^{1/4}$$

Find the 4th roots of $(-5-2i)$

$$(x^4)^{1/4} = (-5-2i)^{1/4}$$



$$25+4 = r^2$$
$$\sqrt{29} = \sqrt{r^2}$$
$$\sqrt{29} = r$$

$$\tan\theta = \frac{+2}{+5}$$
$$\theta = 202^\circ$$

$$\left[\sqrt{29}(\cos 202^\circ + i\sin 202^\circ)\right]^{\frac{1}{4}}$$

$$(\sqrt{29})^{1/4} = (29^{1/2})^{1/4} = 29^{1/8}$$

$$202^\circ \cdot \frac{1}{4} = 50.5^\circ$$

$$360^\circ \cdot \frac{1}{4} = 90^\circ$$

1) $29^{1/8}(\cos 50.5^\circ + i\sin 50.5^\circ)$

2) $29^{1/8}(\cos 140.5^\circ + i\sin 140.5^\circ)$

3) $29^{1/8}\ \text{cis}\ 230.5^\circ$

4) $29^{1/8}\ \text{cis}\ 320.5^\circ$

410.5