

PARTIAL FRACTIONS

$$\left[\frac{15x-14}{x^2-3x+2} dx = \frac{A}{x-1} + \frac{B}{x-2} \right]$$

(x-1) (x-2)

$$15x-14 = Ax - 2A + Bx - B$$

$$15 = A + B$$

$$-14 = -2A - B$$

$$\begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 15 \\ -14 \end{bmatrix} = \begin{bmatrix} -1 \\ 16 \end{bmatrix} \begin{matrix} A \\ B \end{matrix}$$

$$u = x-1 \quad du = dx \quad \int \frac{-1}{x-1} dx + \int \frac{16}{x-2} dx$$

$$\int \frac{-1}{u} du +$$

$$- \ln|u| + 16 \ln|x-2|$$

$$- \ln|x-1| + 16 \ln|x-2| + C$$

$$16 \ln|x-2| - \ln|x-1|$$

$$= \ln \frac{|x-2|^{16}}{|x-1|} + C$$

$$\int \frac{1-x^2}{4x^4+17x^2+4} dx$$

$$\frac{1-x^2}{(4x^2+1)(x^2+4)} = \frac{Ax+B}{4x^2+1} + \frac{Cx+D}{x^2+4}$$

$$-x^2+1 = Ax^3+4Ax+Bx^2+4B+4Cx^3+Cx+4Dx^2+D$$

$$\begin{aligned} 0 &= A + 4C \\ -1 &= B + 4D \\ 0 &= 4A + C \\ 1 &= 4B + D \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & 0 & 4 \\ 4 & 0 & 1 & 0 \\ 0 & 4 & 0 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\int \frac{1/3}{4x^2+1} dx + \int \frac{-1/3}{x^2+4} dx = \begin{bmatrix} 0 \\ 1/3 \\ 0 \\ -1/3 \end{bmatrix}$$

$$\begin{aligned} &= \frac{1}{3} \int \frac{1}{u^2+1} \frac{du}{2} \quad u=2x \quad \frac{1}{3} \cdot \frac{1}{4} \int \frac{1}{\frac{x^2}{4}+1} dx \\ &= \frac{1}{6} \tan^{-1} u \quad \frac{du}{dx} = 2 \quad -\frac{1}{12} \int \frac{1}{u^2+1} 2 du \quad u = \frac{x}{2} \\ &= \frac{1}{6} \tan^{-1}(2x) \quad -\frac{1}{6} \tan^{-1} u + C \quad du = \frac{1}{2} dx \end{aligned}$$

$$= \frac{1}{6} \tan^{-1}(2x) - \frac{1}{6} \tan^{-1}\left(\frac{x}{2}\right) + C$$

$$\frac{19x-32}{(4x+1)(2x-3)^2} = \frac{A}{4x+1} + \frac{B}{(2x-3)^2} + \frac{C}{2x-3}$$

$$\frac{2x+3}{\frac{x^2(x+3)}{(x+0)^2}} = \frac{\frac{A}{x^2}}{(x+0)^2} + \frac{\frac{B}{x^1}}{x^1} + \frac{\frac{C}{x+3}}{x+3}$$

$$\frac{2x^3 + x^2 + 4x - 5}{x^2 + 2x - 3} \quad \text{Long division}$$

$$\begin{array}{r} 2x - 3 \\ x^2 + 2x - 3 \overline{) 2x^3 + x^2 + 4x - 5} \\ \underline{-2x^3 + 4x^2 + 6x} \\ 3x^2 + 2x - 5 \\ \underline{+3x^2 + 6x + 9} \\ 8x - 14 \end{array}$$

$$\int 2x - 3 + \frac{8x - 14}{x^2 + 2x - 3}$$







