TRIGONOMETRIC INTEGRALS

$$\int \sin \underset{A}{4x} \cos \underset{B}{3x}\, dx$$

$$\int e^{4x} dx$$

$$\frac{1}{4} e^{4x}$$

$$\frac{1}{2}\int \left(\sin 7x + \sin x\right) dx$$

$$\frac{1}{2}\left[-\frac{1}{7}\cos 7x - \cos x\right] + C$$

$$-\frac{1}{14}\cos 7x - \frac{1}{2}\cos x + C$$

$$\int \sin^5 x \, dx$$

$$\int \sin x \cdot \sin^4 x \, dx$$

$$\int \sin x \cdot (\sin^2 x)^2 \, dx$$

$$\int \sin x \, (1 - \cos^2 x)^2 \, dx$$

$u = \cos x$

$du = -\sin x \, dx$

$$\int \cancel{\sin x} \underbrace{(1 - u^2)^2}_{(1-u^2)(1-u^2)} \frac{du}{-\cancel{\sin x}}$$

$$-\int (1 - 2u^2 + u^4) \, du$$

$$+\left[-u + \frac{2u^3}{3} - \frac{u^5}{5}\right] + C$$

$$-\cos x + \frac{2}{3}\cos^3 x - \frac{\cos^5 x}{5} + C$$

If one term has an odd power.

1) Split off $\sin x / \cos x$ from the odd power.

2) Rewrite remaining term as $(\sin^2 x)^p$ or $(\cos^2 x)^p$

3) Use $\sin^2 x + \cos^2 x = 1$ to rewrite squared term

4) U-substite, simplify & integrate

$$\int \sin^6 2x \cos^3 2x \, dx$$

$$\int \sin^6 2x \, \underline{\cos^2 2x} \cdot \cos 2x \, dx$$

$$\int \sin^6 2x \left(1 - \sin^2 2x\right) \cos 2x \, dx$$

$u = \sin 2x$

$du = -\cos 2x \cdot 2$

$$\int u^6 \left(1 - u^2\right) \cancel{\cos 2x} \cdot \frac{du}{-2\cancel{\cos 2x}}$$

$$-\frac{1}{2} \int \left(u^6 - u^8\right) du$$

$$= -\frac{1}{2} \left[\frac{u^7}{7} - \frac{u^9}{9} \right] + C$$

$$= \frac{-\sin^7 2x}{14} + \frac{\sin^9 2x}{18} + C$$

$\cos 2x = 1 - 2\sin^2 x$

$\frac{\cancel{2}\sin^2 x}{\cancel{2}} = \frac{1-\cos 2x}{2}$

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$\cos 2x = 2\cos^2 x - 1$

$1 + \cos 2x = 2\cos^2 x$

$\frac{1}{2}(1+\cos \cancel{2x}\,4x) = \cos^2(2x)$

$\int \cos^4 x \, dx$

$\int (\cos^2 x)^2 \, dx$

$\int \left[\frac{1}{2}(1+\cos 2x)\right]^2 dx$

$(1+\cos 2x)(1+\cos 2x)$

$\frac{1}{4}\int(1 + 2\cos 2x + \cos^2 2x)\,dx$

$\frac{1}{4}\int(1+2\cos 2x)\,dx + \frac{1}{4}\int \frac{1}{2}(1+\cos 4x)\,dx$

$\frac{1}{4}\left[x + \cancel{2}\cdot\frac{1}{\cancel{2}}\sin 2x\right] + \frac{1}{8}x + \frac{1}{8}\cdot\frac{1}{4}\sin 4x + C$

$\frac{1}{4}x + \frac{1}{4}\sin 2x + \frac{1}{8}x + \frac{1}{32}\sin 4x + C$

$\frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$

All even powers

1) Rewrite as $(\sin^2 x)^p$ or $(\cos^2 x)^p$
2) Use double identity to rewrite $\sin^2 x$ or $\cos^2 x$
3) FOIL & integrate each term separately.

12) $\int \sin^2 x \cos^4 x \, dx$

$\int \sin^2 x \left(\cos^2 x\right)^2 dx$

$\int \frac{1}{2}(1-\cos 2x)\left[\frac{1}{2}(1+\cos 2x)\right]^2 dx$

$\frac{1}{8}\int (1-\cos 2x)(1+\cos 2x)(1+\cos 2x)\, dx$

$\frac{1}{8}\int (\underset{\#6}{1-\cos^2 2x})(1+\cos 2x)\, dx$

$\frac{1}{8}\int \sin^2 2x\,(1+\cos 2x)\, dx$

$\frac{1}{8}\int \sin^2 2x \, dx + \frac{1}{8}\int \underset{u\text{-sub}}{\sin^2 2x \cos 2x}$

Double angle identity