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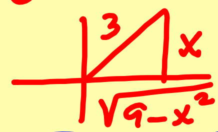
Trig Substitution

$a^2 - x^2$	$x = a\sin\theta$
$x^2 + a^2$	$x = a\tan\theta$
$x^2 - a^2$	$x = a\sec\theta$

$$\int \frac{x^2}{\sqrt{9-x^2}}\,dx \qquad x = 3\sin\theta \qquad dx = 3\cos\theta\,d\theta$$

$$\frac{x}{3} = \sin\theta$$
$$\theta = \sin^{-1}\left(\frac{x}{3}\right)$$



$$\int \frac{9\sin^2\theta}{\sqrt{9-9\sin^2\theta}} \cdot 3\cos\theta\,d\theta$$

$$\frac{27}{3}\int \frac{\sin^2\theta\cos\theta}{\sqrt{1-\underbrace{\sin^2\theta}_{\cos^2\theta}}}\,d\theta$$

$$9\int \frac{\sin^2\theta\,\cancel{\cos\theta}}{\cancel{\cos\theta}}\,d\theta$$

$$9\int \sin^2\theta\,d\theta$$

$$\frac{9}{2}\int (1-\cos 2\theta)\,d\theta$$

$$\frac{9}{2}\left[\theta - \frac{1}{2}\sin 2\theta\right] + C$$

$$\frac{9}{2}\theta - \frac{9}{4}\sin 2\theta + C$$

$$\frac{9}{2}\theta - \frac{9}{4}\sin 2\theta + C$$

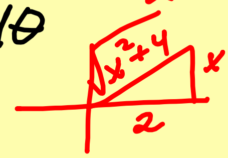
$$\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{9}{4}\cdot 2\sin\theta\cos\theta + C$$

$$\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{\cancel{9}}{2}\left(\frac{x}{\cancel{3}}\right)\left(\frac{\sqrt{9-x^2}}{\cancel{3}}\right) + C$$

$$\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{x\sqrt{9-x^2}}{2} + C$$

$$\int \frac{1}{x^2\sqrt{x^2+4}}\,dx \qquad x = 2\tan\theta \Rightarrow \tan\theta = \frac{x}{2}$$

$$dx = 2\sec^2\theta\, d\theta$$



$$\int \frac{2\sec^2\theta}{4\tan^2\theta\sqrt{4\tan^2\theta+4}}\,d\theta$$

$$\frac{1}{2\cdot 2}\int \frac{\sec^2\theta}{\tan^2\theta\sqrt{\cancel{\tan^2\theta+1}}}\,d\theta \qquad \sec^2\theta$$

$$\frac{1}{4}\int \frac{\sec^{\cancel{2}}\theta}{\tan^2\theta\cdot\cancel{\sec\theta}}\,d\theta$$

$$\frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta$$

$$\frac{1}{4}\int \frac{\frac{1}{\cancel{\cos\theta}}}{\frac{\sin^2\theta}{\cos^2\theta}}\,d\theta \qquad \frac{\cos^2\theta}{\sin^2\theta}$$

$$\frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta \qquad u = \sin\theta,\quad du = \cos\theta\, d\theta$$

$$\frac{1}{4}\int \frac{\cancel{\cos\theta}}{u^2}\cdot\frac{du}{\cancel{\cos\theta}}$$

$$\frac{1}{4}\int \frac{1}{u^2}\,du$$

$$\frac{1}{4}\int u^{-2}\,du$$

$$\frac{1}{4}\cdot\frac{u^{-1}}{-1} + C$$

$$-\frac{1}{4u} + C$$

$$-\frac{1}{4\sin\theta} + C$$

$$\left\{\begin{aligned} &-\frac{1}{4\sin\theta} + C \\ &= \frac{-1}{4\cdot\frac{x}{\sqrt{x^2+4}}} + C \\ &= -\frac{\sqrt{x^2+4}}{4x} + C \end{aligned}\right.$$

$\int \frac{1}{\sec^2\theta} d\theta =$

$\int \cos^2\theta \, d\theta$

$\frac{1}{2}\int (1 + \cos 2\theta) + C$

$\frac{1}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right] + C$

$\int \frac{1}{\csc^2\theta} d\theta$

$\int \sin^2\theta$

$1 + \tan^2\theta = \sec^2\theta$

$\int \tan^2\theta \, d\theta$

$= \int (\sec^2\theta - 1) \, d\theta$

$= \tan\theta - \theta + C$

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$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$$
$$= -\ln|\cos x| + C$$

$$\int \cot x\,dx = \frac{\cos x}{\sin x}$$
$$\ln|\sin x| + C$$

$$\int \sin(nx)\,dx = -\frac{1}{n}\cos(nx) + C$$

$$\int e^{nx}\,dx = \frac{1}{n}e^{nx} + C$$