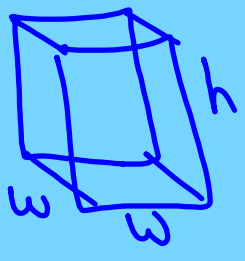
Calculus Semester Review

A shipping container with a square base.

Volume = $250\ ft^3$

Bottom = \$5 Sides = \$3 Top = \$2

What dimensions will minimize the cost?



$C = 5w^2 + 3(4wh) + 2w^2$

$C = 7w^2 + 12wh$

$w^2 h = 250$

$h = \frac{250}{w^2}$

$(0, \infty)$

$\lim_{w \to 0} 7w^2 + \frac{3000}{w} = 0 + \infty = \infty$

$C = 7w^2 + 12w \cdot \frac{250}{w^2}$

$\lim_{w \to \infty} 7w^2 + \frac{3000}{w} \quad \infty + 0 = \infty$

* $C = 7w^2 + \frac{3000}{w} w^{-1}$

5.98	752
6	

$C' = 14w - \frac{3000}{w^2} = 0$

$14w = \frac{3000}{w^2}$

Profit = Revenue − Cost

$\frac{14w^3}{14} = \frac{3000}{14}$

$w^3 =$

$w = 5.98 \approx 6$ $\quad \frac{250}{5.98^2}$

5.98 ft. × 5.98 ft. × 7 ft.

$s(t)$

$v(t) = s'(t)$

$a(t) = v'(t) = s''(t)$

500 ft ahead of van.

Like #18

Go-Kart = Moves at 20 ft/s

Mini Van = accel at 3 ft/sec^2 at rest

$a(t) = 3$

$v(t) = 3t + C$

$0 = 0$

$v(t) = 3t$

$s(t) = \frac{3t^2}{2} + C$

$0 = 0 + C$

$s(t) = \frac{3}{2}t^2$

Go Kart

$v = 20$

$s(t) = 20t + C$

$500 = 0 + C$

$s(t) = 20t + 500$

$\frac{3}{2}t^2 = 20t + 500$

$3t^2 - 40t - 1000 = 0$

$t = 26.1$ sec

$s(26.1) = 20(26.1) + 500$

$= 522 + 500$

$= 1022$ ft.

Hyperbolic Functions

$\frac{d}{dx}\sinh x = \cosh x$

$\frac{d}{dx}\cosh x = +\sinh x$

$\frac{d}{dx}\tanh x = \operatorname{sech}^2 x$

$\frac{d}{dx}\operatorname{csch} x = -\operatorname{csch} x \coth x$

$\frac{d}{dx}\operatorname{sech} x = -\operatorname{sech} x \tanh x$

$\frac{d}{dx}\coth x = -\operatorname{csch}^2 x$

Find $f'(x)$

$$f(x) = (\tanh^2 x)\left(\frac{\ln(\operatorname{sech}^3 x)}{2^{\cosh x}}\right)$$

$$f'(x) = (\tanh^2 x)\left[\frac{2^{\cosh x}\cdot\frac{1}{\operatorname{sech}^3 x}\cdot 3\operatorname{sech}^2 x\cdot -\operatorname{sech} x\tanh x - \ln(\operatorname{sech}^3 x)\cdot \ln 2\cdot 2^{\cosh x}\cdot \sinh x}{\left[2^{\cosh x}\right]^2}\right]$$

$$+\frac{\ln(\operatorname{sech}^3 x)}{2^{\cosh x}}\cdot 2\tanh x\cdot \operatorname{sech}^2 x$$

Integration

Like b. $\int \frac{5x^2 - 4x}{\sqrt[3]{x}}\,dx$

$\int (5x^2 - 4x)\cdot x^{-1/3}\,dx$

$\int (5x^{5/3} - 4x^{2/3})\,dx$

$= \frac{3}{8}5x^{8/3} - \frac{3}{5}4x^{5/3} + C$

$\frac{15}{8}x^{8/3} - \frac{12}{5}x^{5/3} + C$

$\int \tan^5 x \sec^2 x\,dx$

$u = \tan x$

$du = \sec^2 x$

$\int 3x^2 \csc^2(x^3)\,dx$

$u = x^3$

$du = 3x^2\,dx$

$\int 4x^3 \sin^8(x^4)\cos(x^4)\,dx$

$u = \sin(x^4)$

$du = \cos(x^4)\cdot 4x^3$

$\int 4x^3 \cdot u^8 \cos(x^4)\cdot \frac{du}{4x^3 \cos x^4}$

$$\int \frac{4x^2}{\sqrt{9-64x^6}}\,dx$$

(x^3)

$$\frac{4}{3}\int \frac{x^2}{\sqrt{1-\frac{64}{9}x^6}}\,dx$$

$$u = \frac{8}{3}x^3$$

$$du = 8x^2\,dx$$

$$\frac{4}{3}\int \frac{x^2}{\sqrt{1-u^2}}\,\frac{du}{8x^2}$$

$$\frac{1}{6}\int \frac{1}{\sqrt{1-u^2}}\,du$$

$$\frac{1}{6}\sin^{-1}u + C$$

$$\frac{1}{6}\sin^{-1}\left(\frac{8}{3}x^3\right) + C$$

$$\int \cos x \cdot 6^{\sin x}\,dx$$

$$u = \sin x$$

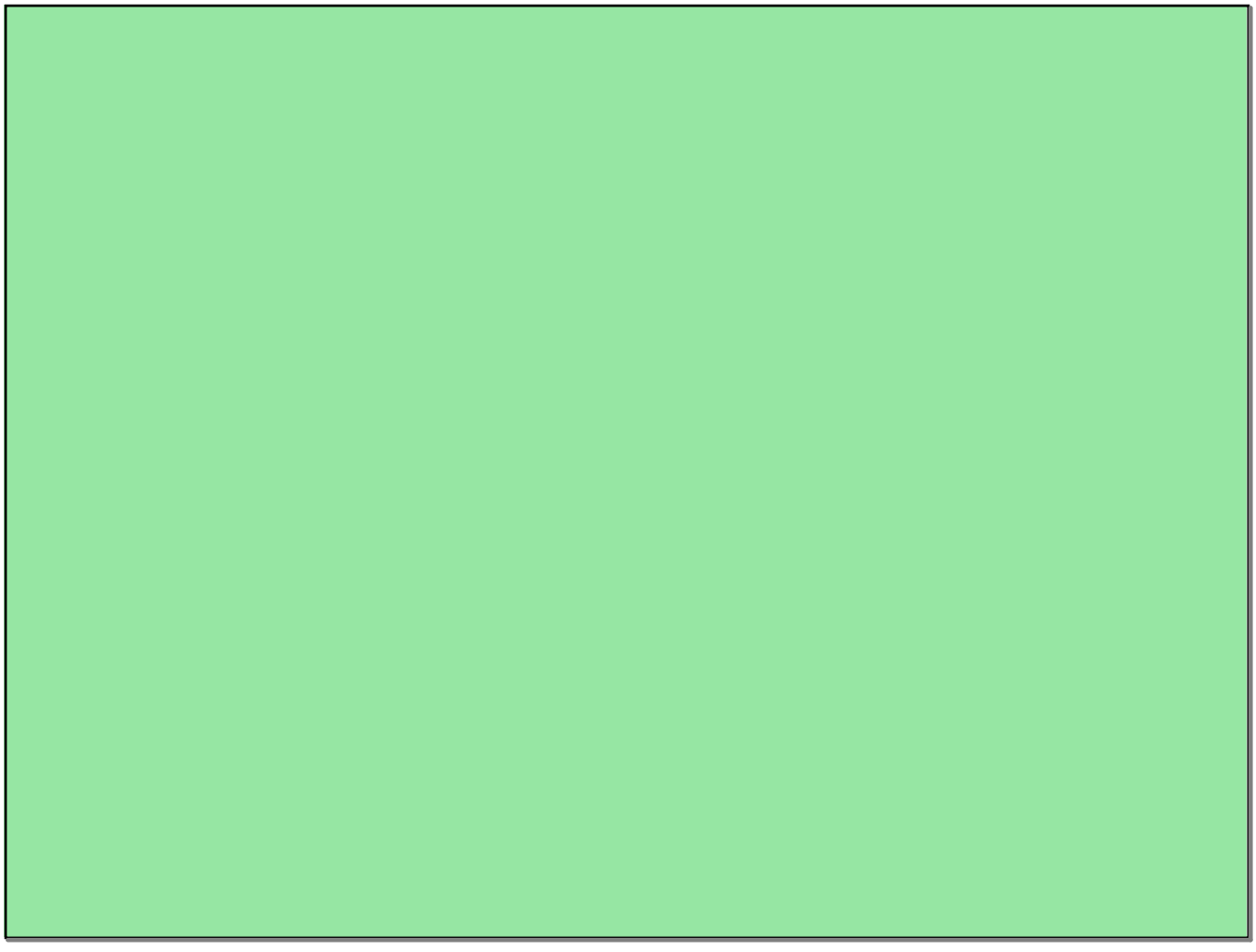
$$du = \cos x\,dx$$

$$\int \cos x \cdot 6^u\,\frac{du}{\cos x}$$

$$\frac{1}{\ln 6}\cdot 6^u + C$$

$$\frac{1}{\ln 6}\,6^{\sin x} + C$$

April 30, 2025



April 30, 2025