

3-VARIABLE ELIMINATION & CRAMER'S RULE

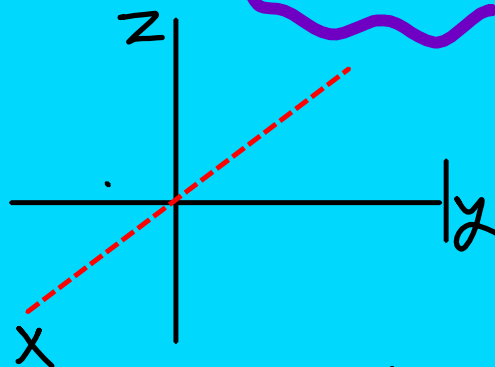
$$\begin{array}{rcl} -3 & \boxed{-x + 8y + 2z = -12} \\ -4 & \boxed{3x + y + 7z = 38} \\ & \boxed{4x - 3y + 6z = 47} \end{array}$$

$$\begin{array}{r} \textcircled{1} \\ -3x - 24y - 6z = 36 \\ 3x + y + 7z = 38 \\ \hline -23y + z = 74 \end{array}$$

$$\begin{array}{r} \textcircled{2} \\ -x - 32y - 8z = 48 \\ 4x - 3y + 6z = 47 \\ \hline -35y - 2z = 95 \end{array}$$

$$\begin{array}{r} \textcircled{3}^2 \boxed{-23y + z = 74} \\ -35y - 2z = 95 \\ \hline -46y + 2z = 148 \\ -35y - 2z = 95 \\ \hline -81y = 243 \\ \frac{-81y}{-81} = \frac{243}{-81} \end{array}$$

$$y = -3$$



$$\begin{array}{l} (2, -3, 5) \\ (-3, 2, -6) \end{array}$$

Where 3 planes intersect

$$\begin{array}{r} \textcircled{4} \\ -23(-3) + z = 74 \\ 69 + z = 74 \\ -69 \quad -69 \\ \hline z = 5 \end{array}$$

$$\begin{array}{r} \textcircled{5} \\ x + 8y + 2z = -12 \\ x + 8(-3) + 2(5) = -12 \\ x - 24 + 10 = -12 \\ x - 14 = -12 \\ \quad \quad \quad \uparrow \text{find} \\ x = 2 \end{array}$$

$$(2, -3, 5)$$

Elimination

- 1) Group 2 equations + eliminate a variable.
- 2) Group a different pair of equations + eliminate the SAME variable.
- 3) Group the 2 resulting equations (with 2 variables) + eliminate a variable.
- 4) Sub answer in a 2-variable equation to find a 2nd variable.
- 5) Sub both answers in a 3-variable equation to get 3rd variable.

CRAMER'S RULE

$$4x + 2y - z = 15$$

$$2x - y + 5z = 9$$

$$3x + 2y - z = 12$$

$$y = \frac{\begin{vmatrix} \textcircled{4} & \textcircled{15} & \textcircled{-1} \\ 2 & 9 & 5 \\ 3 & 12 & -1 \end{vmatrix}}{\begin{vmatrix} 4 & 2 & -1 \\ 2 & -1 & 5 \\ 3 & 2 & -1 \end{vmatrix}}$$

$$x = \frac{\begin{vmatrix} 1 & y_1 & z_1 \\ 2 & y_2 & z_2 \\ 3 & y_3 & z_3 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}}$$

$$y = \frac{\begin{vmatrix} x_1 & 1 & z_1 \\ x_2 & 2 & z_2 \\ x_3 & 3 & z_3 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}}$$

$$4 \begin{vmatrix} 9 & 5 \\ 12 & -1 \end{vmatrix} - 15 \begin{vmatrix} 2 & 5 \\ 3 & -1 \end{vmatrix} + (-1) \begin{vmatrix} 2 & 9 \\ 3 & 12 \end{vmatrix}$$

$$4(-9 - 60) - 15(-2 - 15) - 1(24 - 27)$$

$$-69 \quad -17 \quad -3$$

$$-276 + 255 + 3 = -18$$

$$= 4 \begin{vmatrix} -1 & 5 \\ 2 & -1 \end{vmatrix} - 2 \begin{vmatrix} 2 & 5 \\ 3 & -1 \end{vmatrix} + (-1) \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}$$

$$4(-1 - 10) - 2(-2 - 15) + (-1)(4 - 3)$$

$$-36 \quad +34 \quad -1 = -3$$

$$y = \frac{-18}{-9} = 2$$

Calculator

$$\frac{\det \begin{bmatrix} _ & _ & _ \\ _ & _ & _ \\ _ & _ & _ \end{bmatrix}}{\det \begin{bmatrix} _ & _ & _ \\ _ & _ & _ \\ _ & _ & _ \end{bmatrix}}$$

- 1) Add a fraction from the template menu.
- 2) In numerator, add determinant (Menu-7-3)
- 3) Inside the determinant parentheses, add a 3 x 3 matrix from the template menu.
- 4) Type the numbers into the top matrix.
- 5) Repeat steps 2 & 3, for the bottom determinant.

CRAMER'S RULE

A theater charges:

\$75 Section X

\$55 Section y

\$30 Section Z

$x = \# \text{ of seats in Sec. X}$
 $y = \text{" " " " " " } y$
 $z = \text{" " " " " " } z$

There are 3 times as many
Seats in Section y as Section X.

Revenue for selling all 23,000
Seats is \$870,000.

How many seats in each
Section?

$$3x = y$$

$$3x - y = 0$$

$$y = \frac{\begin{vmatrix} 75 & 870000 & 30 \\ 3 & 0 & 0 \\ 1 & 23000 & 1 \end{vmatrix}}{\begin{vmatrix} 75 & 55 & 30 \\ 3 & -1 & 0 \\ 1 & 1 & 1 \end{vmatrix}}$$

$$75x + 55y + 30z = 870,000$$

$$3x - y + 0z = 0$$

$$x + y + z = 23,000$$