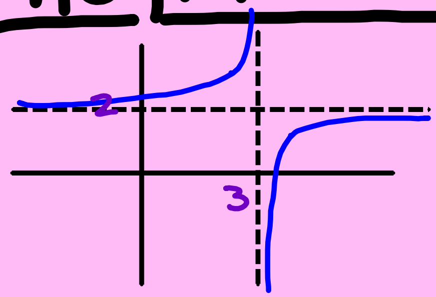
ASYMPTOTES & CONTINUITY



$$f(x) = \frac{-1}{x-3} + \frac{2}{1}\frac{(x-3)}{(x-3)}$$

$$= \frac{-1+2x-6}{x-3}$$

$$\lim_{x\to 3} \frac{2x-7}{x-3} = \frac{1}{0}$$

$$\lim_{x\to 3^+} \frac{2x-7}{x-3} = \frac{-}{+}$$

$$= -\infty$$

$$\boxed{\text{Vertical: } x=3}$$

Horiz

$$\lim_{x\to\infty} \frac{2x-7}{x-3}$$

$$\lim_{x\to\infty} \frac{2\cancel{x}}{\cancel{x}} = 2$$

$$\boxed{\text{Horiz: } y=2}$$

Vertical

$$\lim_{x\to \#} f(x) = \pm\infty$$

↑ where denom = 0

Horizontal

$$\lim_{x\to\pm\infty} f(x) = \#$$

$f(x) = \dfrac{x^2-2x}{x^2+x-6}$

$(x+3)(x-2)$

Vertical

$\lim_{x\to -3^-} \dfrac{x^2-2x}{(x+3)(x-2)} = \dfrac{9+6}{0} = \dfrac{+}{-\cdot-} = +\infty$

$\lim_{x\to 2} \dfrac{x^2-2x}{(x+3)(x-2)} = \dfrac{0}{0}$

$\lim_{x\to 2} \dfrac{x\cancel{(x-2)}}{(x+3)\cancel{(x-2)}} = \dfrac{2}{5}$

Vertical: $x=-3$

Horizontal

$\lim_{x\to\infty} \dfrac{x^2}{x^2} = 1$

Horiz: $y=1$

$$f(x) = \frac{\sqrt{36x^2+11}}{3x-5}$$

Vertical

$$\lim_{x\to 5/3^-} \frac{\sqrt{36x^2+11}}{3x-5} = \frac{\sqrt{31}}{0}$$

$$= \frac{+}{-}$$

$$= -\infty$$

Vertical

$x = 5/3$

Horizontal

$$\lim_{x\to +\infty} \frac{\sqrt{36x^2}}{3x} = \lim_{x\to\infty} \frac{6|x^1|}{3x}$$

$$= \lim_{x\to\infty} \frac{6\cancel{x}}{3\cancel{x}} = 2$$

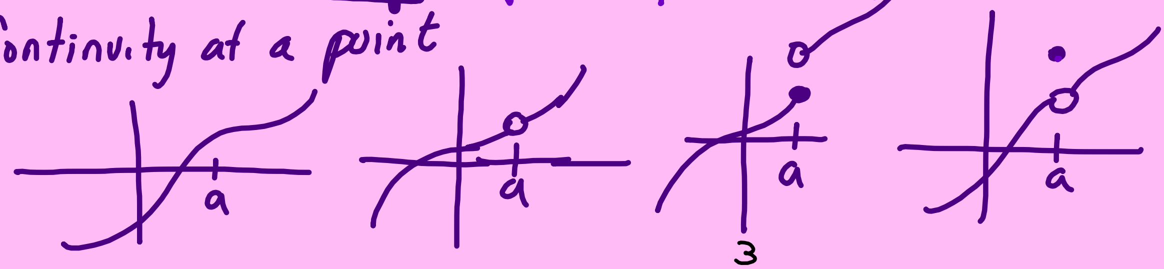
$$\lim_{x\to -\infty} \frac{6|x|}{3x} = \lim_{x\to\infty} \frac{-6\cancel{x}}{3\cancel{x}} = -2$$

Horiz: $y = 2$ + $y = -2$

CONTINUITY

- smooth + unbroken
- no holes, vertical asymp

Continuity at a point



1) $f(a)$ is defined.
2) $\lim_{x \to a} f(x)$ exists.
3) $f(a) = \lim_{x \to a} f(x)$

$$f(x) = \begin{cases} 3x+2 & x<1 \\ 7-2x^2 & x \geq 1 \end{cases}; \quad a=1$$

1) $f(1) = 7 - 2(1)^2 = 5$

2) $\lim_{x \to 1^-} 3x+2 = 5$

$\lim_{x \to 1^+} 7-2x^2 = 5$

$\lim_{x \to 1} f(x) = 5$

3) $f(1) = \lim_{x \to 1} f(x)$

Yes, f is continuous at $a=1$.

$$f(x) = \begin{cases} 3x+8 & x<-3 \\ 4 & x=-3 \\ x^2-10 & x>-3 \end{cases} \qquad a=-3$$

1) $f(-3) = 4$

2) $\lim\limits_{x \to -3^-} 3x+8 = -1$

$\lim\limits_{x \to -3^+} x^2-10 = -1$

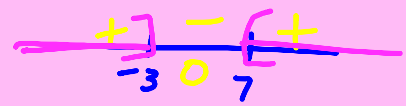
$\lim\limits_{x \to -3} f(x) = -1$

3) $f(-3) \neq \lim\limits_{x \to -3} f(x)$

f is not continuous at -3

$f(x) = \sqrt{x^2-4x-21}$

$(x-7)(x+3)$



$(-\infty, -3]$ C

$(7, \infty)$ C

$(-\infty, 7]$ D

$(-3, 7)$ D