Slope-intercept

$y = mx + b$

Point-Slope

$y - y_1 = m(x - x_1)$

(year, E)
in terms of for

* (1997, 5000)
(2002, 7000)

$m = \frac{7000-5000}{2002-1997} = \frac{2000}{5} = 400$

$y - 5000 = 400(x - 1997)$

$y - 5000 = 400x - 798{,}800$
$+5000$

$y = 400x - 793{,}800$

5(a) $5y + 3x - 10 = 0$

$5y + 3x = 10$

10/3	0
0	2

(10/3, 0)
(0, 2)

$3x + 5y < 15$

5	0
0	3

$0 + 0 < 15$

true

$(x-4)^2$

$y \geq -2|x-4| + 3$

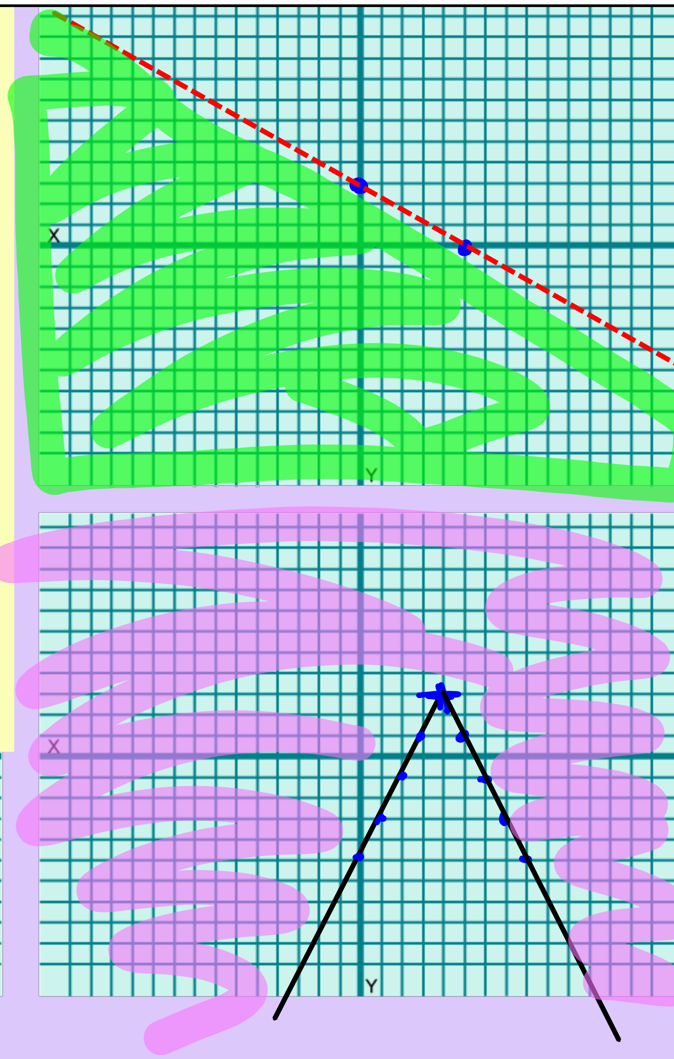
Right

Up 3

$0 \geq -2|0-4| + 3$

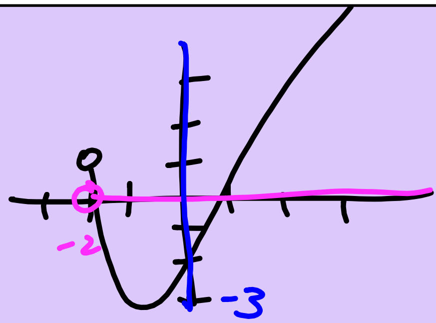
$-8 + 3$

$0 \geq -5$



September 16, 2025

2)



11

$f\left(-\frac{1}{2}\right) = 9$

Domain: $x > -2$

Range: $y \geq -3$

9

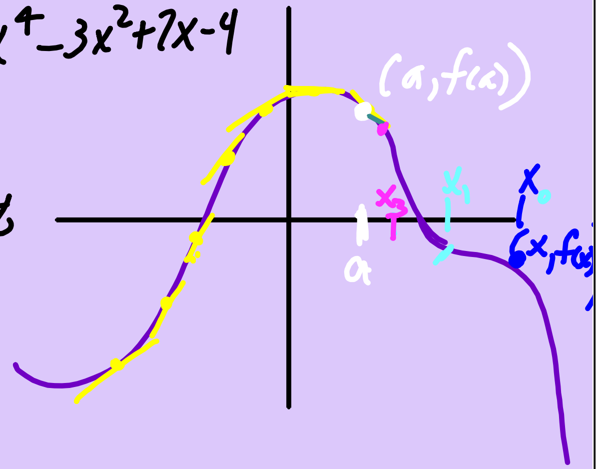
$x < -4$

$-4 \leq x < 2$

$x > 2$

DERIVATIVES

$y = x^4 - 3x^2 + 7x - 4$



- the slope of a line tangent to a curve at a given point

$$m = \frac{f(x) - f(a)}{x - a}$$

$$f'(a) = \lim_{x \to a} \underbrace{\frac{f(x) - f(a)}{x - a}}_{\text{slope}}$$

let's 2 pts. get close to make a point of tangency

1st definition of derivative

Find $f'(a)$ $\quad f(x) = x^3 - 2x^2 + x - 1$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{x \to a} \frac{x^3 - 2x^2 + x - \cancel{1} - (a^3 - 2a^2 + a - \cancel{1})}{x - a}$$

$$\lim_{x \to a} \frac{(x^3 - a^3)(-2x^2 + 2a^2)(x - a)}{x - a}$$

$$\lim_{x \to a} \frac{\cancel{(x-a)}(x^2 + ax + a^2) \overbrace{- 2\cancel{(x-a)}(x+a)}^{-2(x^2-a^2)} + \overbrace{\cancel{x-a}}^{+1(x-a)}}{\cancel{x-a}}$$

$$\lim_{x \to a} x^2 + ax + a^2 - 2x - 2a + 1$$

$$= a^2 + a^2 + a^2 - 2a - 2a + 1$$

$$\boxed{f'(a) = 3a^2 - 4a + 1}$$

$a = -1 \quad 3(-1)^2 - 4(-1) + 1 = 8 = m$

$a = 0 \quad m = 1$

Find eq. of the tangent line at $x = -1$.

$m = f'(-1) = 8$

point $= f(-1) = (-1)^3 - 2(-1)^2 + -1 - 1 = -5$

$(-1, -5)$

$y + 5 = 8(x + 1)$

$y + 5 = 8x + 8$

$\boxed{y = 8x + 3}$

1) find f'
2) Sub # in to get m
3) Get y-coord by subbing # in original $f(x)$

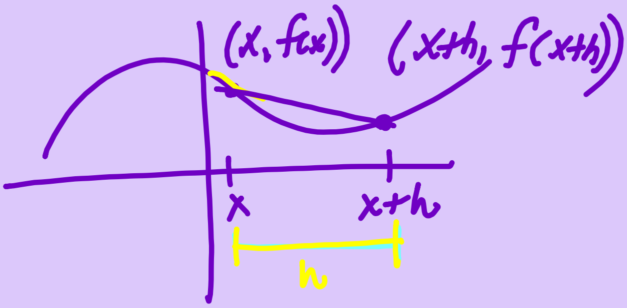
1) $m = f'(\#)$
2) point $f(\#)$
3) Use point-slope

Homework

$f(x) = \frac{1}{x^2}$

$$\lim_{x \to a} \frac{\frac{1}{x^2} - \frac{1}{a^2}}{x - a}$$

Common denom



$$\lim_{h \to 0} \frac{f(x+h)-f(x)}{\cancel{x+h-x}}$$

$$\boxed{\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}}$$

Second def. of deriv.

$f(x) = \sin x$

$$\lim_{x \to a} \frac{\sin(x)-\sin(a)}{x-a}$$

$f(x) = 8\sin x$

$$\lim_{h \to 0} \frac{8\sin(\overset{A+B}{x+h}) - 8\sin x}{h}$$

$$8\lim_{h \to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$8\lim_{h \to 0} \frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h}$$

$$8\lim_{h \to 0} \sin x\frac{(\cos h - 1)}{h} + \cos x \cdot \frac{\sin h}{h}$$

$$8\left[\cancel{\sin x \cdot 0} + \cos x \cdot 1\right]$$

$$= 8\cos x$$

deriv. of the deriv.

	Deriv	2nd deriv.
$f(x)$	$f'(x)$	$f''(x)$
$y=$	$y'=$	y''
$y=$	$\frac{dy}{dx}$	$\frac{d^2}{dx^2}y = \frac{d^2y}{dx^2}$

Gottfried Leibniz
Isaac Newton

September 16, 2025

$\frac{d}{dx}\sin x = \cos x$

$\frac{d}{dx}\tan x = \sec^2 x$

$\frac{d}{dx}\sec x = \sec x \tan x$

$\frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}\cot x = -\csc^2 x$

$\frac{d}{dx}\csc x = -\csc x \cot x$

$f(x) = 4\tan x + 3\csc x$

$f'(x) = 4\sec^2 x - 3\csc x \cot x$

Power Rule

$f(x)$	$f'(x)$
$x^4 - 2x^3 + x - 1$	$4x^3 - 6x^2 + 1$

Power Rule

$$\frac{d}{dx} x^n = n \cdot x^{n-1}$$

$$f(x) = 3x^8 - \frac{1}{4x^5} - 7\sqrt[3]{x^2} + 3$$

$$= 3x^8 - \frac{1}{4}x^{-5} - 7x^{2/3} + 3x^0$$

$$f'(x) = 24x^7 + \frac{5}{4}x^{-6} - \frac{14}{3}x^{-1/3}$$

$$= 24x^7 + \frac{5}{4x^6} - \frac{14}{3x^{1/3}}$$

Find f''.

$$f(x) = x^6 + 3x^9$$

$$f'(x) = 6x^5 + 27x^8$$

$$f''(x) = 30x^4 + 216x^7$$