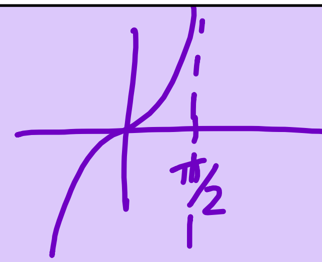
LIMITS at and to ∞

LIMITS

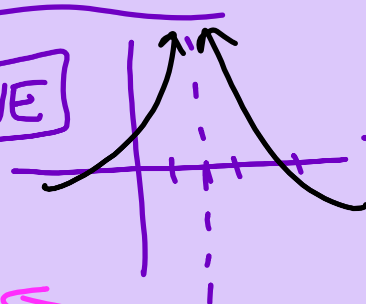
- $x \to \#$
  - Sub # in
    - $\frac{0}{0}$
      - factor
      - conjugate
    - $\frac{\#}{0}$
      - Check one-sided limits for $\pm\infty$ or DNE
- $x \to \pm\infty$
  - Rational func
    - Use highest power in num. & denom
  - Polynomial
    - Sub ∞ in highest power term

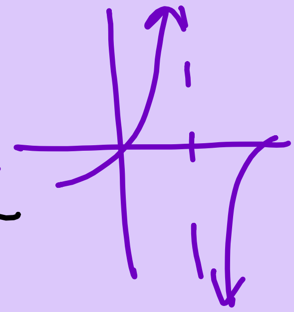
One-Sided Limits

$$\lim_{x\to 5^-} \frac{2x+3}{x-1} = \frac{2(5)+3}{5-1} = \frac{13}{4}$$



$$\lim_{x\to 2} \frac{3x}{x-2} = \frac{6}{0} = \boxed{DNE}$$





$$\lim_{\substack{x\to 2^- \\ 1.9}} \frac{3x}{x-2} = \frac{+}{-} = -\infty$$

Does not agree = DNE

$$\lim_{\substack{x\to 2^+ \\ 2.1}} \frac{3x}{x-2} = \frac{+}{+} = +\infty$$

$$\lim_{x\to 4} \frac{8x^2}{(x-4)^4} = \frac{128}{0} = +\infty$$

These agree, so

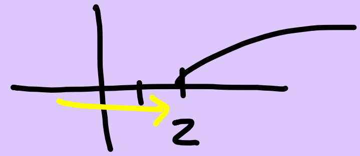
$$\lim_{\substack{x\to 4^- \\ 3.9}} \frac{8x^2}{(x-4)^4} = \frac{+}{+} = +\infty$$

$$\lim_{\substack{x\to 4^+ \\ 4.1}} \frac{8x^2}{(x-4)^4} = \frac{+}{+} = +\infty$$

$$\lim_{x\to -3^+} \frac{8x}{(x+3)^2} = \frac{-24}{0} = \frac{-}{+} = \boxed{-\infty}$$

-2.9

$$\lim_{x\to 2^-} \sqrt{x-2} = \cancel{\emptyset} = \text{DNE}$$



$$f(x) = \begin{cases} \dfrac{2x+1}{3x-1} & x < -1 \\ \dfrac{1}{(x-1)^2} & x > -1 \end{cases}$$

$$\lim_{x\to -1} f(x) = \frac{1}{4}$$

$$\lim_{x\to -1^-} \frac{2x+1}{3x-1} = \frac{+1}{+4}$$

$$\lim_{x\to -1^+} \frac{1}{(x-1)^2} = \frac{1}{4}$$

What if

$$\lim_{x\to -1^+} \frac{1}{(x+1)^2} = \frac{1}{0} = \frac{+}{+} = +\infty$$

-0.9

Limits to $\pm\infty$

$$\lim_{x\to\infty} \frac{x^2 \cdot \frac{1}{x^2}}{7x^2+3 \cdot \frac{1}{x^2}} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{\frac{\cancel{x^2}}{\cancel{x^2}}}{\frac{7\cancel{x^2}}{\cancel{x^2}}+\frac{3}{x^2}}$$

$$\lim_{x\to\infty} \frac{1}{7+\frac{3}{x^2}\to 0} = \frac{1}{7+0} = \boxed{\frac{1}{7}}$$

$$\frac{3}{\infty^2}$$

$$\lim_{x\to\infty} \frac{\cancel{x^2}}{7\cancel{x^2}} = \frac{1}{7}$$

$$\lim_{y \to -\infty} \frac{5y^3+4}{3y+7} = \lim_{y \to -\infty} \frac{5\cancel{y^3}^{2}}{3\cancel{y}}$$

$$= \lim_{y \to -\infty} \frac{5y^2}{3} = \frac{5}{3}(-\infty)^2 = +\infty$$

$$\lim_{y \to -\infty} \frac{3y}{5y^3} = \lim_{y \to \infty} \frac{3}{5y^2} = \frac{3}{\infty} = 0$$