MORE CHAIN RULE

$f(x) = \cos(3x^2 - 7x)$

$f'(x) = -\sin(3x^2 - 7x) \cdot (6x - 7)$

$f(x) = (\cos x)(3x^2 - 7x)$

product rule

$f(x) = \tan^8(x^5 - 3x^4)$

$= [\tan(x^5 - 3x^4)]^8$

$f'(x) = 8\tan^7(x^5 - 3x^4)\sec^2(x^5 - 3x^4) \cdot (5x^4 - 12x^3)$

$f(x) = \sec(8x^2)$

$f'(x) = \sec(8x^2)\tan(8x^2) \cdot 16x$

$f(x) = \csc^5(\cot(3x^7)) = [\csc(\cot(3x^7))]^5$

$f'(x) = 5\csc^4(\cot(3x^7)) \cdot -\csc(\cot(3x^7))\cot(\cot(3x^7)) \cdot$
$-\csc^2(3x^7) \cdot 21x^6$

$f(x) = \underline{\csc^5(x^2)} \cdot \underline{\cot(4x^8)}$

$f'(x) = \csc^5(x^2) \cdot -\csc^2(4x^8) \cdot 32x^7 + \cot(4x^8) \cdot$

$5\csc^4(x^2) \cdot -\csc(x^2)\cot(x^2) \cdot 2x$

$f(x) = \tan\left(\sec(x^4 - 2x)^6\right)$

$f'(x) = \sec^2\left(\sec(x^4 - 2x)^6\right) \cdot \sec(x^4 - 2x)^6 \tan(x^4 - 2x)^6$

$\cdot\ 6(x^4 - 2x)^5 \cdot (4x^3 - 2)$

DIFFERENTIALS

$y = f(x)$

$\frac{\Delta y}{\Delta x} = \frac{dy}{dx} = f'(x)$

$dy = f'(x)\,dx$

Find dy.

$y = x^3 - 3x^2 + 7$

$\frac{dy}{dx} = 3x^2 - 6x$

$dy = (3x^2 - 6x)\,dx$

The radius of a sphere is measured to be 20 in. with a possible error of ±0.3 in.
Estimate the possible error in volume.

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$dV = 4\pi r^2\, dr$$

$$dV = 4\pi(20)^2 \cdot \pm(0.3)$$

$$\approx \pm 150.8 \text{ in}^3$$

% error

% error of radius

$$\frac{\text{change}}{\text{original}} = \frac{dr}{r} = \frac{\pm 0.3 \cancel{\text{in}}}{20 \cancel{\text{in}}}$$

$$= 0.015$$

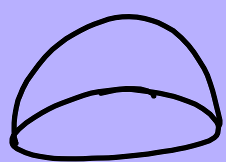
$$\approx 1.5\%$$

% error of volume

$$\frac{dV}{V} = \frac{\cancel{4\pi r^2}\, dr}{\cancel{\frac{4}{3}\pi r^2}\, r} = 3\frac{dr}{r}$$

$$= 3(1.5\%)$$

$$= 4.5\%$$



Hemisphere
$r = 12$ ft.

Coat with paint $= 0.002$ ft thick

Estimate the volume of paint.

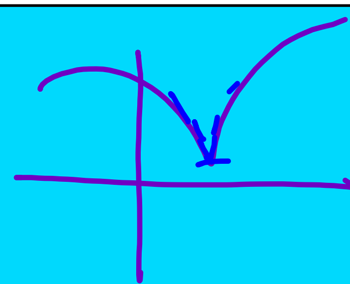
$$V = \frac{2}{3}\pi r^3$$

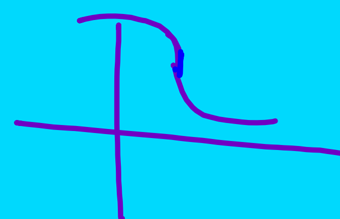
$$dV = 2\pi r^2\, dr$$

$$= 2\pi (\underset{ft}{12})^2 (\underset{ft}{0.002}) = 1.81 \text{ ft}^3$$

DIFFERENTIABILITY

* must first be continuous
* no sudden changes in slope
  no sharp pts
  no breaks
  no vertical tangent lines





1) $f(a)$ is defined.
2) $\lim_{x \to a} f(x)$ exists.
3) $f(a) = \lim_{x \to a} f(x)$
4) $f'(a)^- = f'(a)^+$

$$f(x) = \begin{cases} \sqrt{x} & \text{if } x \geq 4 \\ 6-x & \text{if } x < 4 \end{cases} \quad ; \quad a = 4$$

1) $f(4) = \sqrt{4} = 2$

2) $\lim_{x \to 4^-} (6-x) = 6-4 = 2$

$\lim_{x \to 4^+} \sqrt{x} = \sqrt{4} = 2$

$\lim_{x \to 4} f(x) = 2$

3) $f(4) = \lim_{x \to 4} f(x)$

Yes, f is continuous

4) $f'(x)^- = -1$

$f'(4)^- = -1$

$f'(x)^+ = \frac{1}{2} x^{-1/2} = \frac{1}{2\sqrt{x}}$

$f'(4)^+ = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

$f'(\cancel{x}4)^- \neq f'(4)^+$

not differentiable