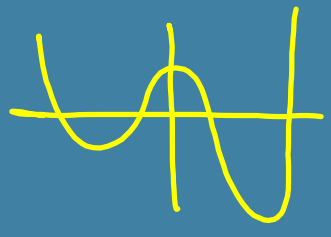
Polynomials & Rational Functions

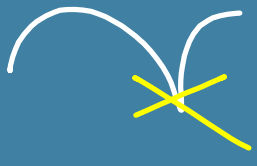
Polynomials – one or more terms
– whole # exponents

$x^4 - 3x^3 + 2x^2 = 7$



Graph Characteristics

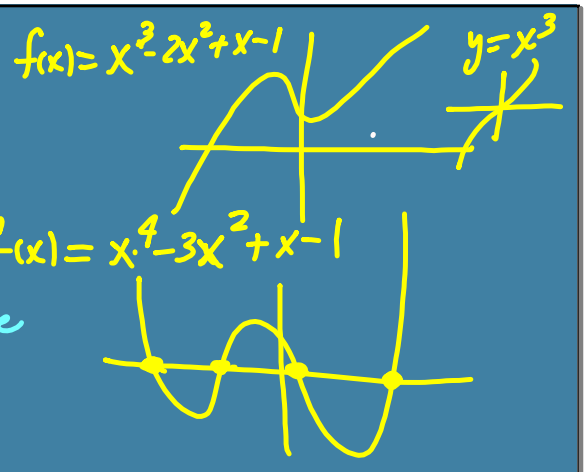
1) Smooth, rounded turns
2) Continuous (no asymptotes, no holes, no sharp points, no end pts.)



Degree – highest power

Max # of relative max/min = Degree − 1

Max # of x-intercepts (Roots, Zeros): Degree



End Behavior

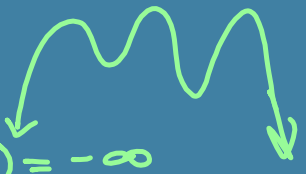
even degree – both ends go in same direction

odd degree – ends go in opposite directions

$f(x) = -3x^6 + 7x^5 - 28x^3 + 2x^2 - 7$

$\lim_{x \to -\infty} f(x) = -\infty$ (x: far to the left; y)

$\lim_{x \to +\infty} f(x) = -\infty$ far to the right



Max of Relative Max/Min: $6 - 1 = 5$

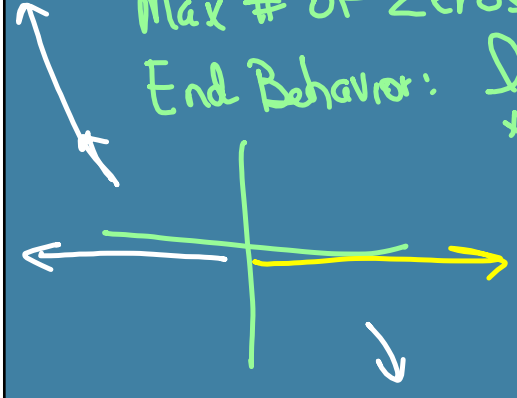
Max of Zeros: 6

$f(x) = -2x^5 + 7x^3 - 4x + 1$

Max # of relative extrema: $5 - 1 = \boxed{4}$

Max # of zeros? 5

End Behavior: $\lim_{x \to -\infty} f(x) = +\infty$ $\lim_{x \to +\infty} f(x) = -\infty$



Solving Polynomials

$(2x^3 - 8x^2)(+3x - 12) = 0$

1) $2x^2(x-4) + 3(x-4)$

2) $(x-4)(2x^2+3) = 0$

$x-4=0$ $\quad$ $2x^2+3=0$

$x=4$ $\quad$ $\sqrt{x^2} = \sqrt{\frac{-3}{2}}$

$x = \frac{\pm i\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$x = \frac{\pm i\sqrt{6}}{2}$

Find the zeros by calculator
Menu - Analyze Graph - Zero

Write the equation of the polynomial with roots: $-2, \frac{2}{5}, +4$.

Work as a reverse factoring problem.

$x = -2$ $\quad$ $x = \frac{2}{5}$ $\quad$ $x = 4$

$x+2=0$ $\quad$ $5x=2$ $\quad$ $x-4=0$

$5x-2=0$

$(x+2)(5x-2)(x-4) = 0$

$(x+2)(5x^2 - 22x + 8) = 0$

$5x^3 - 22x^2 + 8x + 10x^2 - 44x + 16$

$5x^3 - 12x^2 - 36x + 16 = 0$

Solve.

$2x^5 + 3x^4 - x^3 + 9x^2 - 55x - 30 = 0$

$(x)(x-2)(x)(x)(x)$

$\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30$

$\pm 1, \pm 2$

	2	3	−1	9	−55	−30
2		+ 4	14	26	70	30
	2	7	13	35	15	0

$(x-2)(2x^4 + 7x^3 + 13x^2 + 35x + 15)$

$\pm 1, \pm 15, \pm 3, \pm 5$

	2	7	13	35	15
−3		−6	−3	−30	−15
	2	1	10	5	0

$(x-2)(x+3)((2x^3 + x^2)(+10x + 5))$

$(x-2)(x+3)[x^2(2x+1) + 5(2x+1)]$

$(x-2)(x+3)(2x+1)(x^2+5) = 0$

$x = 2, -3, -\frac{1}{2}, \pm i\sqrt{5}$

$x^2 + 5 = 0$

$\sqrt{x^2} = \pm\sqrt{5}$

$= \pm i\sqrt{5}$

Steps

1) Find a number that makes $f(x) = 0$ using factors of last term / factors of leading coeff.
2) Use # to perform synthetic division
3) Repeat if needed
4) Solve all factors.