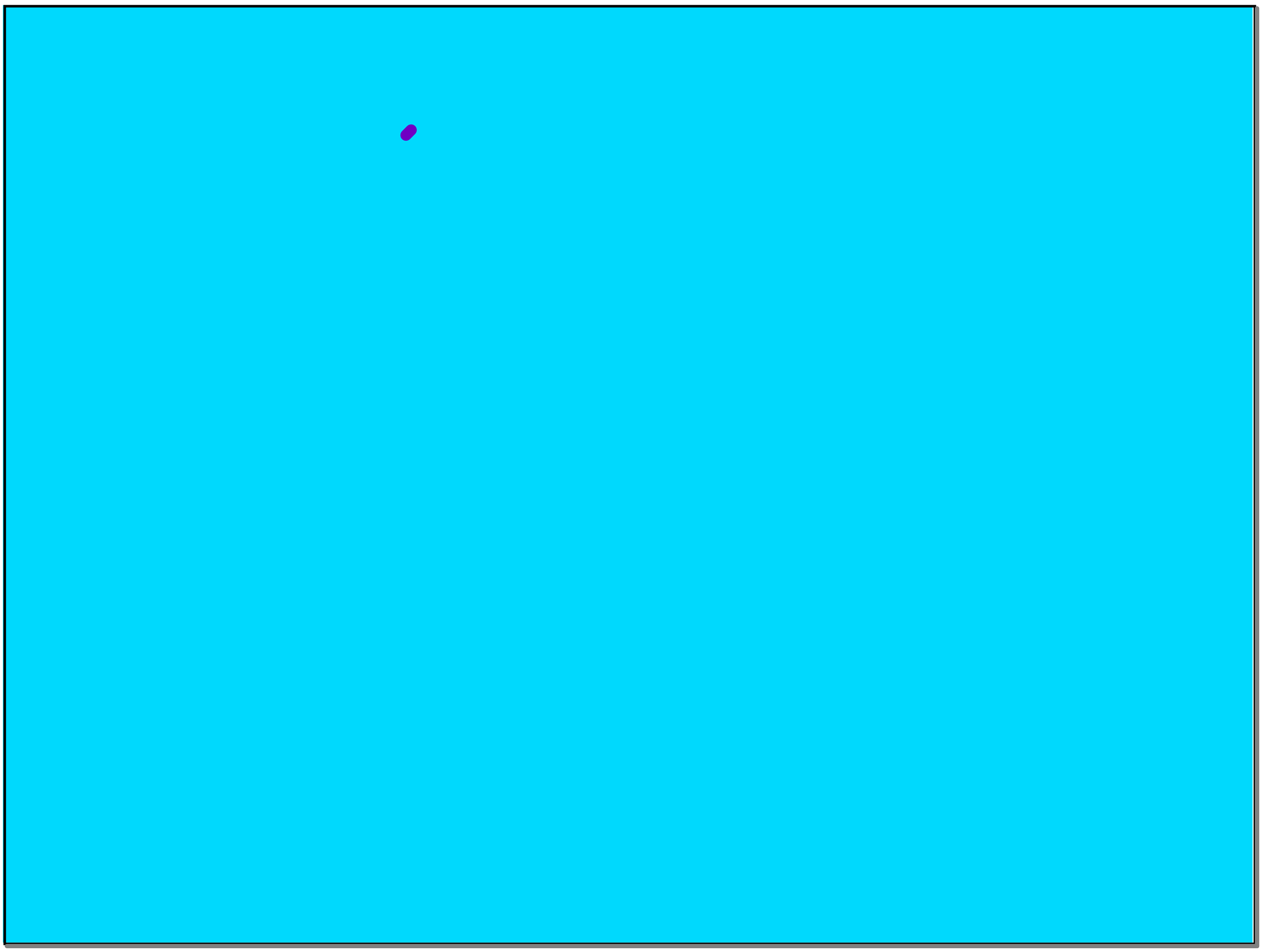
September 22, 2025



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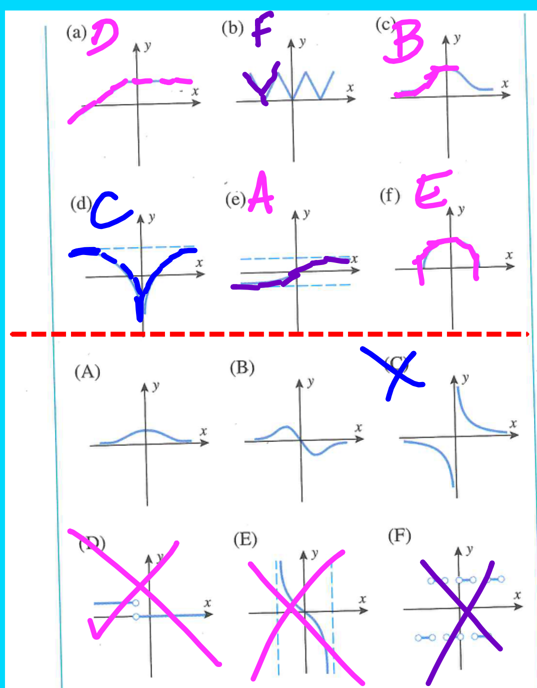
Product, Quotient, & Chain Rules



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Product, Quotient, & Chain Rules

$f(x) = 3x^2 \cdot 4x^5 = 12x^7$ $\qquad f'(x) = 84x^6$

$f'(x) = \cancel{6x \cdot 20x^4 = 120x^5}$

$3x^2 \cdot 20x^4 + 4x^5 \cdot 6x$

$= 60x^6 + 24x^6 = 84x^6$

Product Rule

$$\frac{d}{dx} f \cdot g = f \cdot g' + g \cdot f'$$

1st · d'2nd + 2nd · d'1st

$f(x) = (x^6 - 3x^8 + 7)(3x^{-4} + 2x^{1/3} - 5)$

$f'(x) = (x^6 - 3x^8 + 7)\left(-12x^{-5} + \frac{2}{3}x^{-2/3}\right) + (3x^{-4} + 2x^{1/3} - 5)(6x^5 - 24x^7)$

QUOTIENT RULE

$$f(x) = \frac{1}{x} = x^{-1}$$

$$f'(x) = -1\,x^{-2} = \frac{-1}{x^2}$$

$$\frac{d}{dx}\,\frac{f}{g} = \frac{g \cdot f' - f \cdot g'}{g^2}$$

$$= \frac{\text{low} \cdot d'\text{high} - \text{high} \cdot d'\text{low}}{\text{low}^2}$$

$$f(x) = \frac{4\tan x - 3x^5}{8x^{2/9} - \csc x}$$

$$f'(x) = \frac{(8x^{2/9} - \csc x)(4\sec^2 x - 15x^4) - (4\tan x - 3x^5)\left(\frac{16}{9}x^{-7/9} + \csc x \cot x\right)}{(8x^{2/9} - \csc x)^2}$$

$$f(x) = \frac{\tan x \sec x}{\cos x}$$

$$f'(x) = \frac{\cos x\left[\overset{\text{1st}}{\tan x} \cdot \overset{d'\text{2nd}}{\sec x \tan x} + \overset{\text{2nd}}{\sec x} \cdot \overset{d'\text{1st}}{\sec^2 x}\right] - \tan x \sec x \cdot -\sin x}{\cos^2 x}$$

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CHAIN RULE

$$\frac{d}{dx} f[g(h(x))] = f'[g(h(x))] \cdot g'(h(x)) \cdot h'(x)$$

x^8

$8x^7$

$$f(x) = (7x^9 - 3x^5)^8$$

$$f'(x) = 8(7x^9 - 3x^5)^7 \cdot (63x^8 - 15x^4)$$

$$f(x) = \sqrt[4]{(x^7 - 5x)(x^4 + 9x^2)^3} = [\underbrace{(x^7 - 5x)(x^4 + 9x^2)^3}_{\text{product}}]^{1/4}$$

$$f'(x) = \frac{1}{4}[(x^7 - 5x)(x^4 + 9x^2)^3]^{-3/4} \cdot [\underbrace{(x^7 - 5x)}_{\text{1st}} \cdot \underbrace{3(x^4 + 9x^2)^2 \cdot (4x^3 + 18x)}_{\text{d' 2nd}} +$$

$$\underbrace{(x^4 + 9x^2)^3}_{\text{2nd}} \cdot \underbrace{(7x^6 - 5)}_{\text{d'1st}}]$$

Find eq of tangent line at $x = -1$

$m = f'(-1)$

$f(-1) = y$