

# RADICAL EQUATIONS

Solve.

$$\sqrt{3x+4} - \sqrt{x+2} = 2$$

$$(\sqrt{3x+4})^2 = (2 + \sqrt{x+2})^2 (2 + \sqrt{x+2})$$

$$3x+4 = 4 + 2\sqrt{x+2} + 2\sqrt{x+2} + x+2$$

$$3x+4 = 6 + x + 4\sqrt{x+2}$$

$$\begin{array}{r} -x \quad -6 \\ (2x-2)^2 = (4\sqrt{x+2})^2 \end{array}$$

$$(2x-2)(2x-2)$$

$$4x^2 - 8x + 4 = 16(x+2)$$

$$4x^2 - 8x + 4 = 16x + 32$$

$$4x^2 - 24x - 28 = 0$$

$$4(x^2 - 6x - 7) = 0$$

$$4(x-7)(x+1) = 0$$

$$x=7, \quad \cancel{x=-1}$$

check in  
original  
problem!

$$x=7 \quad \sqrt{25} - \sqrt{9} = 2$$

$$5 - 3 = 2$$

$$\checkmark 2 = 2$$

$$x=-1 \quad \sqrt{1} - \sqrt{1} = 2$$

$$1 - 1 \neq 2$$

# PARTIAL FRACTIONS

purpose  
to find the fractions  
original that were  
combined to  
make the  
given  
fraction.

$$\frac{(x-4)2}{(x-4)x+3} + \frac{6(x+3)}{x-4(x+3)} = \frac{2x-8+6x+18}{(x+3)(x-4)} = \frac{8x+10}{(x+3)(x-4)} = \frac{8x+10}{x^2-x-12}$$

$$\frac{8x+10}{x^2-x-12}$$

$$\frac{(x-4)}{(x+3)} \left[ \frac{8x+10}{(x-4)(x+3)} = \frac{A}{\cancel{x-4}} + \frac{B}{\cancel{x+3}} \right]$$

$$8x+10 = A(x+3) + B(x-4)$$

$$8x+10 = \underline{Ax} + \underline{3A} + \underline{Bx} - \underline{4B}$$

$$\begin{aligned} 8 &= A + B \\ 10 &= 3A - 4B \end{aligned}$$

$$\begin{bmatrix} 1 & 1 \\ 3 & -4 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 8 \\ 10 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

$$\boxed{\frac{6}{x-4} + \frac{2}{x+3}}$$

$$\frac{\quad}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$$

$$\frac{\quad}{(x^2+4)(x^3+7)} = \frac{Ax+B}{x^2+4} + \frac{Cx^2+Dx+E}{x^3+7}$$

$$\frac{\quad}{(x-5)^2(x+3)} = \frac{A}{(x-5)^2} + \frac{B}{x-5} + \frac{C}{x+3}$$

$$\frac{\quad}{x^3(4x+1)} = \frac{\frac{A}{x^3}}{(x-0)^3} + \frac{\frac{B}{x^2}}{(x-0)^2} + \frac{\frac{C}{x}}{(x-0)^1} + \frac{\frac{D}{4x+1}}{(x-0)^0}$$

$$\frac{10x^2 + 24x + 8}{(x^3 + 3x^2 + 4x + 12)}$$

$$x^2(x+3) + 4(x+3)$$

$$\left[ \frac{10x^2 + 24x + 8}{(x+3)(x^2+4)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+4} \right]$$

$$10x^2 + 24x + 8 = A(x^2+4) + (Bx+C)(x+3)$$

$$10x^2 + 24x + 8 = Ax^2 + 4A + Bx^2 + 3Bx + Cx + 3C$$

$$\begin{aligned} 10 &= A+B \\ 24 &= 3B+C \\ 8 &= 4A+3C \end{aligned} \quad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 1 \\ 4 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 24 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 8 \\ 0 \end{bmatrix}$$

$$\boxed{\frac{2}{x+3} + \frac{8x}{x^2+4}}$$

If:

$$0x^2 + 4x + 9 = Ax^2 + Bx - 4B + Cx - 5C$$

$$0 = A$$

$$4 = B + C$$

$$9 = -4B - 5C$$

