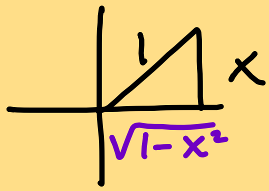
Inverse Trig Functions

$y = \sin^{-1} x$



$a^2 + x^2 = 1$

$\sqrt{a^2} = \sqrt{1 - x^2}$

$\frac{x}{1} = \sin y$

$1 = \cos y \frac{dy}{dx}$

$\frac{1}{\cos y} = \frac{dy}{dx}$

$\frac{\frac{1}{\sqrt{1-x^2}}}{1} = \frac{dy}{dx}$

$\frac{1}{\sqrt{1-x^2}} = \frac{dy}{dx}$

$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$ $\qquad$ $\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx} \tan^{-1} x = \frac{1}{x^2+1}$ $\qquad$ $\frac{d}{dx} \cot^{-1} x = \frac{-1}{x^2+1}$

$\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$ $\qquad$ $\frac{d}{dx} \csc^{-1} x = \frac{-1}{|x|\sqrt{x^2-1}}$

$$f(x) = \sin^{-1}(7x^5) \qquad \frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$

$$f'(x) = \frac{1}{\sqrt{1-(7x^5)^2}} \cdot 35x^4$$

$$= \frac{35x^4}{\sqrt{1-49x^{10}}}$$

$$f(x) = \csc^{-1}(x^4)\,\tan^{-1}(\ln x^2)$$

$$f'(x) = \csc^{-1}(x^4) \cdot \frac{1}{(\ln x^2)^2+1} \cdot \frac{1}{x^2} \cdot 2x + \tan^{-1}(\ln x^2) \cdot \frac{-1}{|x^4|\sqrt{(x^4)^2-1}} \cdot 4x^3$$

$$= \frac{2\csc^{-1}(x^4)}{x\left((\ln x^2)^2+1\right)} - \frac{4x^3\tan^{-1}(\ln x^2)}{x^4\sqrt{x^8-1}}$$

L'Hopital's Rule

Indeterminate Forms

$\frac{0}{0}, \frac{\infty}{\infty}$

$\lim_{x \to 2} \frac{x^2-4}{x-2} = \frac{0}{0}$

$\lim_{x \to 2} \frac{(x+2)(x-2)}{x-2} = 4$

L'Hop

$\lim_{x \to 2} \frac{2x}{1} = 4$

L'Hopital's Rule

If $\frac{0}{0}$ or $\frac{\infty}{\infty}$:

$\lim_{x \to \#} \frac{f(x)}{g(x)} = \lim_{x \to \#} \frac{f'(x)}{g'(x)}$

$\lim_{x \to 1} \frac{x^3-3x^2+5x-3}{x^2+x-2} = \frac{1-3+5-3}{1+1-2} = \frac{0}{0}$

$\lim_{x \to 1} \frac{3x^2-6x+5}{2x+1} = \frac{3-6+5}{2+1} = \frac{2}{3}$

$$\lim_{x\to 0} \frac{e^x - 1 - x}{\cos(2x) - 1} = \frac{1-1-0}{1-1} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{e^x - 1}{-\sin(2x)\cdot 2} = \frac{1-1}{0\cdot 2} = \frac{0}{0}$$

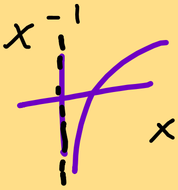
$$\lim_{x\to 0} \frac{e^x - 1}{-2\sin(2x)}$$

$$\lim_{x\to 0} \frac{e^x}{-2\cos(2x)\cdot 2} = \frac{1}{-2\cdot 1\cdot 2} = -\frac{1}{4}$$

$$\lim_{x\to 0^+} \frac{1-\ln x}{e^{1/x}}$$

$$= \frac{1+{+\infty}}{e^{1/0}} = \frac{\infty}{}$$

x^{-1}



$\lim_{x\to -\infty} e^x = 0$ $\quad \lim_{x\to \infty} e^x = +\infty$

$\lim_{x\to 0^+} \ln x = -\infty$ $\quad \lim_{x\to \infty} \ln x = +\infty$

$\frac{1}{x} = x^{-1}$ $$= \frac{\infty}{e^{+\infty}} \to \frac{\infty}{\infty}$$

$$\lim_{x\to 0^+} \frac{-\frac{1}{x}}{e^{1/x}\cdot -1x^{-2}}$$

$$\lim_{x\to 0^+} \frac{-\frac{1}{x}}{\frac{-e^{1/x}}{x^2}} = \lim_{x\to 0^+} -\frac{1}{x}\cdot\frac{x^2}{e^{1/x}}$$

$$= \lim_{x\to 0^+} \frac{-x}{e^{1/x}}$$

$$= \frac{0}{\infty}$$

$$= \boxed{0}$$

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