41
a
49
~~15~~
~~23~~

15 $f(x) = \ln\left(\frac{x+1}{x-1}\right)$

$$f'(x) = \frac{1}{\frac{x+1}{x-1}} \cdot \left[\frac{\cancel{(x-1)}\cdot 1 - \cancel{(x+1)}\cdot 1}{(x-1)^2}\right]$$

$$= \frac{\cancel{x-1}}{x+1}\left[\frac{-2}{(x-1)^{\cancel{2}}}\right]$$

$$= \frac{-2}{(x+1)(x-1)} \text{ OR } \frac{-2}{x^2-1}$$

~~23~~ $y = 8^x$

$$\frac{dy}{dx} = 8^x \cdot \ln 8$$

47) $h(x) = x^{\sqrt{x}}$ $\qquad a = 4$

$$= x^{x^{1/2}}$$

$$= e^{\ln x^{x^{1/2}}}$$

$$= e^{x^{1/2}\cdot \ln x}$$

$$h'(x) = e^{x^{1/2}\cdot \ln x}\cdot\left[x^{1/2}\cdot\frac{1}{x} + \ln x\cdot\frac{1}{2}x^{-1/2}\right]$$

$$= x^{\sqrt{x}}\cdot x^{-1/2}\left[\cancel{x}\cdot\frac{1}{\cancel{x}} + \frac{1}{2}\ln x\right]$$

$$= x^{\sqrt{x}}\cdot\frac{1}{\sqrt{x}}$$

$$= \frac{x^{\sqrt{x}}}{\sqrt{x}}\left[1 + \frac{1}{2}\ln x\right]$$

OR

$$\frac{x^{\sqrt{x}}}{2\sqrt{x}}\left[2 + \ln x\right]$$

$$\frac{4^{\sqrt{4}}}{2\sqrt{4}}\left[2 + \ln 4\right]$$

$$\frac{16}{4}$$

$$= 4\left[2 + \ln 4\right]$$

$$= 8 + 4\ln 4$$

49/ $f(x) = (\sin x)^{\ln x}$ $\qquad a = \frac{\pi}{2}$

$$= e^{\ln x \cdot \ln(\sin x)}$$

$$f'(x) = e^{\ln x \cdot \ln(\sin x)} \left[\ln x \cdot \frac{1}{\sin x} \cdot \cos x + \ln(\sin x) \right]$$

$$= (\sin x)^{\ln x} \left[\frac{\ln x \cdot \frac{\cos x}{\sin x}}{\frac{1}{x}} + \frac{\ln(\sin x)}{x} \right]$$

$$\frac{(\sin x)^{\ln x}}{x} \left[x \ln x \cdot \cot x + \ln(\sin x) \right]$$

$$= \frac{\left(\sin \frac{\pi}{2}\right)^{\ln \pi/2}}{\pi/2} \left[\frac{\pi}{2} \ln\left(\frac{\pi}{2}\right) \cdot \cot \pi/2 + \ln\left(\sin \frac{\pi}{2}\right) \right]$$

$$= \frac{1^{\ln(\pi/2)}}{\pi/2} \left[\frac{\pi}{2} \ln\left(\frac{\pi}{2}\right) \cdot 0 + \ln(1) \right]$$

$$= \frac{1}{\pi/2} \left[0 + 0 \right]$$

$$= \frac{2}{\pi} \cdot 0$$

$$= 0$$

(a) Find $\frac{dy}{dx}$.

$$y + \ln(xy) = 1$$

$$1\frac{dy}{dx} + \frac{1}{xy}\cdot\left[x\cdot 1\frac{dy}{dx} + y\cdot 1\right] = 0$$

$$\frac{dy}{dx} + \frac{1}{y}\frac{dy}{dx} + \frac{1}{x} = 0$$

$$\frac{dy}{dx}\left(1 + \frac{1}{y}\right) = -\frac{1}{x}$$

$$\frac{dy}{dx}\left(\frac{y+1}{y}\right) = -\frac{1}{x}$$

$$\frac{dy}{dx} = -\frac{1}{x}\cdot\frac{y}{y+1}$$

$$\frac{dy}{dx} = \frac{-y}{x(y+1)}$$