SPECIAL DERIVATIVES

Implicit Diffentation

Explicit

$y = 3x^2 + 7x - 4$

$\frac{dy}{dx} = 6x + 7$

Implicit

$y^2 + 3xy + 7 = 2 - 5y$

Find $\frac{dy}{dx}$ ← "normal"

$y^2 + x^3 + y^3 = 5$

Pretend

$y = 3x^2 + 7x - 4$

$\frac{dy}{dx} = 6x + 7$

$(3x^2+7x-4)^2 + x^3 + (3x^2+7x-4)^3 = 5$

$2(3x^2+7x-4)^1 \cdot (6x+7) + 3x^2 + 3(3x^2+7x-4)^2 \cdot (6x+7) = 0$

$2y \cdot \frac{dy}{dx} + 3x^2 + 3y^2 \frac{dy}{dx} = 0$

$2y \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = -3x^2$

$\frac{dy}{dx}(2y + 3y^2) = -3x^2$

$$\boxed{\frac{dy}{dx} = \frac{-3x^2}{2y + 3y^2}}$$

Find $\frac{dy}{dx}$.

$$(3x^2)(y^3) + 4y^5 = 6\sin y + 8x^5$$

$$3x^2 \cdot 3y^2\frac{dy}{dx} + y^3 \cdot 6x + 20y^4\frac{dy}{dx} = 6\cos y\frac{dy}{dx} + 40x^4$$

$$9x^2y^2\frac{dy}{dx} + 6xy^3 + 20y^4\frac{dy}{dx} = 6\cos y\frac{dy}{dx} + 40x^4$$

$$\frac{dy}{dx}\left(9x^2y^2 + 20y^4 - 6\cos y\right) = 40x^4 - 6xy^3$$

$$\boxed{\frac{dy}{dx} = \frac{40x^4 - 6xy^3}{9x^2y^2 + 20y^4 - 6\cos y}}$$

Find the eq. of the tangent line at $(1,0)$

$$m = \frac{40(1)^4 - 6(1)(0)^3}{9(1)^2(0)^2 + 20(0)^4 - 6\cos 0}$$

$$= \frac{40 - 0}{0 + 0 - 6(1)}$$

$$m = \frac{40}{-6} = -\frac{20}{3}$$

$$y - 0 = \frac{-20}{3}(x - 1)$$

$$\boxed{y = -\frac{20}{3}x + \frac{20}{3}}$$

Find $\frac{dx}{dy}$ ← "normal"

$$\frac{x^2}{y} = 4y^3 + 6x$$

$$\frac{y \cdot 2x\frac{dx}{dy} - x^2 \cdot 1}{y^2} = 12y^2 + 6\frac{dx}{dy}$$

$$\cancel{y^2}\left[\frac{2xy\frac{dx}{dy} - x^2}{\cancel{y^2}} = 12y^2 + 6\frac{dx}{dy}\right]$$

$$2xy\frac{dx}{dy} - x^2 = 12y^4 + 6y^2\frac{dx}{dy}$$

$$\frac{dx}{dy}\left(2xy - 6y^2\right) = 12y^4 + x^2$$

$$\frac{dx}{dy} = \frac{12y^4 + x^2}{2xy - 6y^2}$$

October 2, 2025

Find $\frac{da}{dp}$.

normal

$$3r^7+6a^5-4p=p^7$$

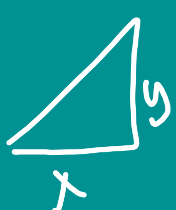
$$21r^6\frac{dr}{dp}+30a^4\frac{da}{dp}-4=7p^6$$

$$\frac{\cancel{30a^4}\frac{da}{dp}}{\cancel{30a^4}}=\boxed{\frac{7p^6+4-21r^6\frac{dr}{dp}}{30a^4}}$$

Find $\frac{dy}{dt}$.

$$4x^2+2y^5=\cos x$$

$$8x\frac{dx}{dt}+10y^4\frac{dy}{dt}=-\sin x\frac{dx}{dt}$$



$$10y^4\frac{dy}{dt}=-8x\frac{dx}{dt}-\sin x\frac{dx}{dt}$$

$$\frac{dy}{dt}=\frac{-8x\frac{dx}{dt}-\sin x\frac{dx}{dt}}{10y^4}$$