Natural Log Operations

$$\ln x + \ln(x+3) = 2$$

$$e^{\ln(x^2+3x)} = e^2$$

$$x^2+3x = e^2$$

$$x^2+3x-e^2 = 0$$

$$x = \frac{-3 \pm \sqrt{9-4(1)(-e^2)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9+4e^2}}{2}$$

$$x = 1.605, \cancel{-4.605}$$

$$\frac{2e^{2x-5}}{2} = \frac{32}{2}$$

$$\ln(e^{2x-5}) = \ln(16)$$

$$2x-5 = \ln(16)$$

$$\frac{2x}{2} = \frac{\ln(16)+5}{2}$$

$$x = 3.886$$

$4^{2x+3} = 7^{x-1}$

$\log 4^{2x+3} = \log 7^{x-1}$ ← log

$(2x+3)\log 4 = (x-1)\log 7$ ← plog

$2x\log 4 + 3\log 4 = x\log 7 - \log 7$

$2x\log 4 - x\log 7 = -3\log 4 - \log 7$

$x(2\log 4 - \log 7) = -3\log 4 - \log 7$

$x = \dfrac{-3\log 4 - \log 7}{2\log 4 - \log 7} \approx -7.385$

$e^{2x} + 3e^x = 28$

$e^{2x} + 3e^x - 28 = 0$

$(e^x + 7)(e^x - 4) = 0$

$e^x + 7 = 0 \quad e^x - 4 = 0$

$\ln e^x = \ln(-7) \quad \ln e^x = \ln 4$

$x = \ln(-7)$ ~~$x = \ln(-7)$~~ $x = \ln 4$

≈ 1.386

Radioactive Iodine has a half-life of 60 days It is considered to be safe when 5% or less is left. How many days will it take to reach a safe level.

$N = N_0 e^{kt}$

$0.5 = 1\, e^{K \cdot 60}$

$\ln 0.5 = \ln e^{60K}$

$\frac{\ln(0.5)}{60} = \frac{60K}{60}$

$-0.0116 = K$

$0.05 = 1\, e^{-0.0116t}$

$\ln(0.05) = \ln(e^{-0.0116t})$

$\frac{\ln(0.05)}{-0.0116} = \frac{-0.0116t}{-0.0116}$

$258 \text{ days} = t$

Newton's Law of Cooling

$u = T + (u_0 - T)e^{kt}$

u: final temp, T: air temp, u_0: initial temp, air

Room Temp = 71°

Normal body temp = 98.6

Body found = 75°

1 hr. later = 72°

$72 = 71 + (75-71)e^{k \cdot 1}$

$-71 \quad -71$

$\frac{1}{4} = \frac{4e^k}{4}$

$\ln 1/4 = \ln e^k$

$\ln(1/4) = k$

$-1.386 = k$

$75 = 71 + (98.6-71)e^{-1.386t}$

$-71 \quad -71$

$\frac{4}{27.6} = \frac{27.6e^{-1.386t}}{27.6}$

$\ln\left(\frac{4}{27.6}\right) = \ln e^{-1.386t}$

$\frac{\ln\left(\frac{4}{27.6}\right)}{-1.386} = \frac{-1.386t}{-1.386}$

$1.39 \text{ hrs} = t$