October 21, 2025

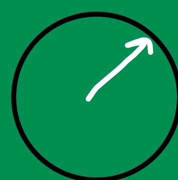
Related Rates

rate of one part of the situation impacts the rate of another part.

$\frac{mi}{h}$ $\frac{m}{s}$ $\frac{\ell}{min}$ $\frac{ft^3}{min}$

$\frac{bushels}{hr}$

Example 1



$r = 0.02\,\frac{in}{sec}$

$\frac{\Delta A}{\Delta t} = \frac{dA}{dt}$

$$\frac{d}{dt}\left[A = \pi r^2\right]$$

$$1\frac{dA}{dt} = 2\pi r\,\frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(4\ in)\left(0.02\,\frac{in}{s}\right)$$

$$\frac{dA}{dt} = 0.16\pi$$

$$\approx \boxed{0.5\,\frac{in^2}{sec}}$$

1) Draw a picture.
2) Label changing parts with variables + unchanging parts with constants.
3) Set up a formula that relates the changing values.
4) Use implicit differentiation with respect to time to find derivative of both sides
5) Identify the rate to be found.
6) Fill in values + Solve.

2) 

$$\frac{dV}{dt} = 0.2\,\frac{m^3}{min}$$

$$S.A = 0.64\pi\ m^2$$

$$\frac{d}{dt}\left[V = \frac{4}{3}\pi r^3\right]$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$-0.2 = 4\pi(0.4)^2 \frac{dr}{dt}$$

$$\frac{\frac{m^3}{min}}{m^2} \quad \frac{-0.2}{0.64\pi} = \frac{0.64\pi}{0.64\pi}\frac{dr}{dt}$$

$$-0.0994 = \frac{dr}{dt}$$

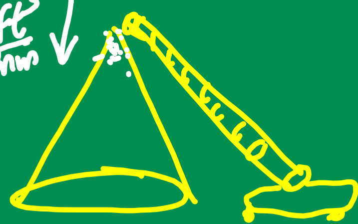
$$-0.1\,\frac{m}{min}$$

$$\frac{4\pi r^2}{4\pi} = \frac{0.64\cancel{\pi}}{4\cancel{\pi}}$$

$$\sqrt{r^2} = \sqrt{0.16}$$

$$r = 0.4$$

3 $\quad 5\,\frac{ft^3}{min}\downarrow$



$h = 2r$

$h = 10$ ft.

Find $\frac{dh}{dt}$.

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 \cdot h$$

$$V = \frac{1}{3}\pi \cdot \frac{h^2}{4} \cdot h$$

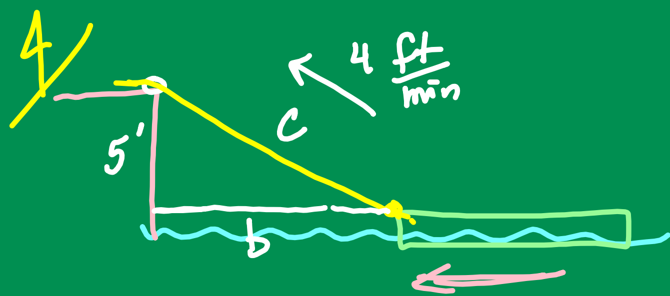
$$\frac{d}{dt}\left[V = \frac{\pi}{12} h^3\right]$$

$$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$$

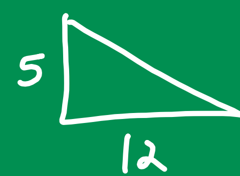
$$5 = \frac{\pi}{4}(10)^2 \frac{dh}{dt}$$

$$\frac{ft^3}{min}\; 5 = 25\pi\, ft^2 \frac{dh}{dt}$$

$$\frac{1}{5\pi}\frac{ft}{min} = \frac{dh}{dt}$$



barge = 12 ft from dock



$$5^2 + 12^2 = c^2$$
$$169 = c^2$$
$$13 = c$$

$$5^2 + b^2 = c^2$$

$$\frac{d}{dt}\left[25 + b^2 = c^2\right]$$

$$0 + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$$

$$2(\underset{ft}{12})\frac{db}{dt} = 2(\underset{ft}{13})(-4 \tfrac{ft}{min})$$

$$\underset{ft}{24}\frac{db}{dt} = \underset{\frac{ft^2/min}{ft}}{-104}$$

$$\frac{db}{dt} = \frac{-104}{24} = \frac{-13}{3}\frac{ft}{min}$$